

MA 214 002 Calculus IV (Spring 2016)

Answers to Review Problems for Final Exam (A)

1. (a)  $f(t) = 1 - H(t-1) + (t^2 - 2t + 2)(H(t-1) - H(t-2)) + 2H(t-2) = 1 + (t-1)^2 H(t-1) - (t-2)^2 H(t-2) - 2(t-2)H(t-2);$   
 $F(s) = \frac{1}{s} + \frac{2}{s^3} e^{-s} - \frac{2}{s^3} e^{-2s} - \frac{2}{s^2} e^{-2s}.$ 

(b)  $f(t) = -\sin t (H(t-\pi) - H(t-2\pi)) = \sin(t-\pi)H(t-\pi) + \sin(t-2\pi)H(t-2\pi);$   
 $F(s) = \frac{1}{s^2 + 1} (e^{-\pi s} + e^{-2\pi s}).$
2. (a)  $y(t) = e^t/4 - e^{-t}/4 - te^{-t}/2.$   
 (b)  $y(t) = \frac{1}{2}e^{-t} \sin 2t - H(t-1)e^{-(t-1)} \sin 2(t-1).$
3. (a)  $2e^{2t} \cos \sqrt{2}t + \frac{3\sqrt{2}}{2}e^{2t} \sin \sqrt{2}t.$   
 (b)  $\frac{2}{7} \left( \frac{1}{\sqrt{2}} \sin \sqrt{2}t - \frac{1}{4} \sin 4t \right).$   
 (c)  $-\frac{1}{s^2 + 1} e^{-\pi s/2}.$   
 (d)  $e^6 \cdot \frac{1}{s-3} \cdot e^{-2s}.$   
 (e)  $\frac{1}{s^2 + 1} \cdot \frac{1}{(s+1)^2}.$
4.  $x(t) = 2e^t - e^{t/2} \cos \left( \frac{\sqrt{3}}{2}t \right) + \frac{\sqrt{3}}{3}e^{t/2} \sin \left( \frac{\sqrt{3}}{2}t \right).$
5. (a) The recurrence relation is:

$$c_{n+2} = -\frac{n-2}{n+2}c_n \quad \text{for } n = 0, 1, 2, \dots.$$

$$y_1 = 1 + x^2; \quad y_2 = x + \frac{1}{3}x^3 - \frac{1}{5 \cdot 3}x^5 + \frac{1}{7 \cdot 5}x^7 - \dots.$$

- (b) The recurrence relation is:

$$c_{n+2} = \frac{2-n}{(n+2)(n+1)}c_n \quad \text{for } n = 0, 1, 2, \dots.$$

$$y_1 = 1 + x^2; \quad y_2 = x + \frac{1}{3!}x^3 - \frac{1}{5!}x^5 + \frac{3}{7!}x^7 - \dots.$$