MA 214 002 Calculus IV (Spring 2016)

Review Problems for Final Examination (A)

1. Express each of the following functions in terms of the Heaviside function and find its Laplace transform:

   (a) $f(t) = \begin{cases} 1 & \text{if } 0 \le t < 1 \\ t^2 - 2t + 2 & \text{if } 1 \le t < 2 \\ 2 & \text{if } 2 \le t < \infty \end{cases}$

   (b) $f(t) = \begin{cases} 0 & \text{if } 0 \le t < \pi \\ -\sin t & \text{if } \pi \le t < 2\pi \\ 0 & \text{if } 2\pi \le t < \infty \end{cases}$

2. Use the Laplace transform to solve the following initial value problems:

   (a) $y''' + y'' - y' - y = 0, \quad y(0) = y'(0) = 0, \quad y''(0) = 1.$

   (b) $y'' + 2y' + 5y = -2t^2\delta(t-1), \quad y(0) = 0, \quad y'(0) = 1.$

3. Evaluate the following Laplace and inverse Laplace transforms:

   (a) $\mathcal{L}^{-1}\{(2s-1)/((s^2-4s+6)\}$;

   (b) $\mathcal{L}^{-1}\left\{\dfrac{4}{(s^2+2)(s^2+16)}\right\}$;

   (c) $\mathcal{L}\{H(t-\frac{\pi}{2})\cos t\}$;

   (d) $\mathcal{L}\{e^{3t}H(t-2)\}$;

   (e) $\mathcal{L}\{\sin t * te^{-t}\}$.

4. Use the Laplace transform to solve the integral equation

$$x(t) = \int_0^t \cos(t-\tau)x(\tau)d\tau + e^t.$$

5. For the equations given in (a) and (b) below, find two power-series solution y_1 and y_2 of the form $y = \sum_{n=0}^{\infty} c_n x^n$ that satisfies the conditions

$$y_1(0) = 1, \quad y_1'(0) = 0 \qquad \text{and} \qquad y_2(0) = 0, \quad y_2'(0) = 1.$$

   For each solution, give the first four non-trivial terms (unless the series terminates sooner). In your work identify the recurrence relation for the coefficients.

   (a) $(1+x^2)y'' - 2y = 0$;

   (b) $y'' + xy' - 2y = 0$.