MA 214 002 Calculus IV (Spring 2016)

Review Problems for Final Examination (B)

1. (a) Show that the following equation is exact and solve the given initial value problem:

$$(x^3 + \frac{y}{x}) + (y^2 + \ln x)\frac{dy}{dx} = 0, \qquad y(1) = 2, \qquad x > 0.$$

   (b) Solve the following initial value problem:

$$ty' = -3y + \frac{\sin t}{t^2}, \qquad y(\pi) = 1.$$

   (c) Determine the longest interval on which the solution of the initial value problem

$$(1 - x^2)\frac{dy}{dx} = 2y, \qquad y(0) = 1$$

   is defined, and solve the initial value problem.

2. A 400-gal tank initially contains 100 gal of brine with 50 lb of salt dissolved in it. At time $t = 0$, brine containing 1 lb and 2 lb of salt per gallon enters the tank from two sources at the rate of 3 gal/s and 1 gal/s, respectively. The well-mixed brine in the tank flows out at the rate of 3 gal/s.

   (a) Set up the initial-value problem that governs the mass of salt in the tank from $t = 0$ to the instant $t = T$ when the tank just begins to overflow.

   (b) Determine the mass of salt in the tank as a function of t for the time interval $[0, T]$.

   (c) Determine the mass of salt in the tank as a function of t for $t \geq T$.

3. Use the method of undetermined coefficients to find a particular solution for each of the following differential equations:

   (a) $y'' + 2y' + y = 3te^{-t} + 2e^{-t}$;

   (b) $y'' - 2y' + 5y = e^t \cos 3t$.

   Write down the general solution for each of the given equations.

4. Determine the *form* of a particular solution y_p for each of the following differential equations if the method of undetermined coefficients were used to solve it.

   (a) $y'' + 9y = x^2 \sin 3x + \cos 2x$;

   (b) $y'' + 2y' + 2y = e^{-x} \cos x + e^x \sin x$.

   You need *not* actually compute the coefficients in y_p.

5. (a) Verify that $y_1 = t$ is a solution of the homogeneous equation

$$t^2y'' + ty' - y = 0, \qquad \text{for } t > 0.$$

(b) Use the method of reduction of order to find a second solution y_2 of the given equation.

(c) Show that $\{y_1, y_2\}$ constitutes a fundamental set of solutions of the given homogeneous equation.

(d) Use the method of variation of parameters to find the general solution of the nonhomogeneous equation

$$t^2y'' + ty' - y = t^{1/2}, \qquad \text{for } t > 0.$$