MA 214 Calculus IV (Spring 2016)

Sections 2

Review Problems 2

1. (a) Verify that $y_1(t) = t$ is a solution of the homogeneous differential equation

$$t^2 y'' - ty' + y = 0$$

on the interval $(0, \infty)$.

(b) Use the method of reduction of order to find a second independent solution y_2.

(c) Find the general solution of the given differential equation.

2. Use the method of undetermined coefficients to find a particular solution for each of the following differential equations:

(a) $y'' - y' = t + e^t$;

(b) $y'' + y' - 2y = \sin t + t\, e^t$.

3. Determine the *form* of a particular solution $Y(x)$ for each of the following differential equations if the method of undetermined coefficients were used to solve it.

(a) $$y'' - 4y' + 4y = x^2 + 2xe^{2x} + x \sin 2x;$$

(b) $$y'' + 2y' + 2y = 3x^2 e^{-x} \cos 2x + xe^{-x} \sin 2x + 1.$$

You need not actually compute the coefficients in $Y(x)$.

4. Use the method of variation of parameters to find a particular solution of the differential equation

$$y'' - 2y' + y = x^{-1} e^x. \qquad (x > 0)$$

Decide whether the method of undetermined coefficients could also have been used. Justify your answer.

5. (a) Show that there are two values of r for which $y(x) = x^r$ satisfies the differential equation

$$x^2 y'' - 2y = 0$$

for $x > 0$. Hence find a fundamental set of solutions of the preceding equation in the interval $(0, \infty)$. [Hint: substitute $y(x) = x^r$ into the given equation and solve for r.]

(b) Find the general solution of the differential equation

$$x^2y'' - 2y = x^2$$

for $x > 0$.

6. (a) Determine μ, R, and δ so as to write the expression

$$u = e^{-t/4}(-\cos 2t + \sin 2t)$$

in the form $u = Re^{-t/4}\cos(\mu t - \delta)$.

(b) A small object of mass 1 kg is attached to a spring with spring-constant 5 N/m and is immersed in a viscous medium with damping constant 2 N·s/m. At time $t = 0$, the mass is lowered 1/4 m and released from rest.

i. Formulate the initial-value problem that governs the displacement u of the small object from its equilibrium position. Specify the sign convention in your definition of u.

ii. Solve the initial-value problem you formulated.

7. Evaluate the following Laplace and inverse Laplace transforms:

(a) $\mathcal{L}[f(t)]$, where $f(t) = 0$ for $0 \le t < 1$ and $f(t) = t$ for $t \ge 1$;

(b) $\mathcal{L}^{-1}[(2s-3)/(s^2+2s+10)]$;

(c) $\mathcal{L}^{-1}[5/((s^2+3)(s^2+25))]$

8. Use the Laplace transform to solve the following initial value problems:

(a) $y''' + y'' - y' - y = 0, \quad y(0) = y'(0) = 0, \quad y''(0) = 1.$

(b) $y'' + 3y' + 2y = \sin 2t, \quad y(0) = 0, \quad y'(0) = 1.$