MA 214 Calculus IV (Spring 2016)

Section 2

Review Problems 2 (Answers)

1. (a) Let $L[y] = t^2y'' - ty' + y$. Then $L[t] = 0 - t + t = 0$. Therefore $y_1(t) = t$ is a solution of the given homogeneous differential equation.

   (b) Using the method of reduction of order, we find that $y_2(t) = t\ln t$ is a second solution. Since $W(y_1, y_2) = t > 0$ for $t \in (0, \infty)$, y_1 and y_2 constitute a fundamental set of solutions on the interval $(0, \infty)$.

   (c) The general solution of the given differential equation is $y = c_1t + c_2t\ln t$, where c_1 and c_2 are arbitrary constants.

2. (a) $Y(t) = -\frac{1}{2}t^2 - t + te^t$.

   (b) $Y(t) = -\frac{1}{10}\cos t - \frac{3}{10}\sin t + t(\frac{1}{6}t - \frac{1}{9})e^t$.

3. (a) $Y(x) = Ax^2 + Bx + C + x^2(Dx + E)e^{2x} + (Fx + G)\sin 2x + (Hx + I)\cos 2x$.

   (b) $Y(x) = (Ax^2 + Bx + C)e^{-x}\cos 2x + (Dx^2 + Ex + F)e^{-x}\sin 2x + G$.

4. A particular solution of the given equation is given by $Y(x) = xe^x\ln x$. The method of undetermined coefficients cannot be used, because $g(x) = x^{-1}e^x$ is not of a form for which that method can handle.

5. (a) Substituting $y(x) = x^r$ into the given equation, we obtain the quadratic equation $r^2 - r - 2 = 0$, which has two solutions $r_1 = -1$ and $r_2 = 2$. Hence $y_1 = 1/x$ and $y_2 = x^2$ are solutions of the given differential equation. Since $W(y_1, y_2) = 3$, they constitute a fundamental set of solutions on the interval $(0, \infty)$.

   (b) The general solution of the given differential equation is:

$$y = \frac{c_1}{x} + c_2x^2 + \frac{1}{3}x^2\ln x.$$

6. (a) $\mu = 2$, $R = \sqrt{2}$, $\delta = 3\pi/4$ or $\tan^{-1}(-1) + \pi$.

   (b) i. Let u be the displacement of the mass from its equilibrium position, with the sign convention that downward is positive. The initial-value problem in question is:

$$u'' + 2u' + 5u = 0, \qquad u(0) = 1/4, \quad u'(0) = 0.$$

ii. The solution of the initial-value problem is:

$$u = \frac{1}{4}e^{-t}\cos 2t + \frac{1}{8}e^{-t}\sin 2t.$$

7. (a) $\mathcal{L}[f(t)] = \frac{e^{-1}}{s}\left(1 + \frac{1}{s}\right)$ for $s > 0$.

   (b) $\mathcal{L}^{-1}[(2s-3)/(s^2+2s+10)] = 2e^{-t}\cos 3t - \frac{5}{3}e^{-t}\sin 3t$.

   (c) $\mathcal{L}^{-1}[\frac{5}{(s^2+3)(s^2+25)}] = \frac{1}{22}\left(\frac{1}{\sqrt{3}}\sin\sqrt{3}t - \frac{1}{5}\sin 5t\right)$.

8. (a) $y(t) = \frac{1}{4}e^{t} - \frac{1}{4}e^{-t} - \frac{1}{2}te^{-t}$.

   (b) $y(t) = \frac{7}{5}e^{-t} - \frac{5}{4}e^{-2t} - \frac{3}{20}\cos 2t - \frac{1}{20}\sin 2t$.