Newton's Approximation of Pi

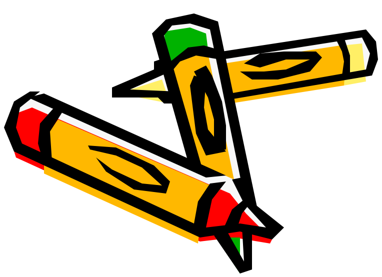
By: Sarah Riffe
and Jen Watt

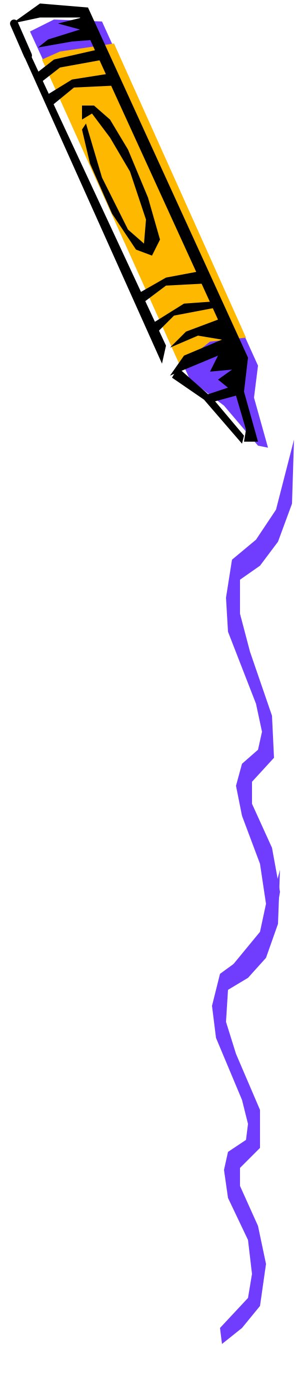
Outline

- Who was Isaac Newton? What was his life like?

- What is the history of Pi?

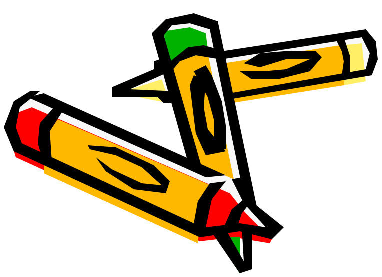
- What was Newton’s approximation of Pi?



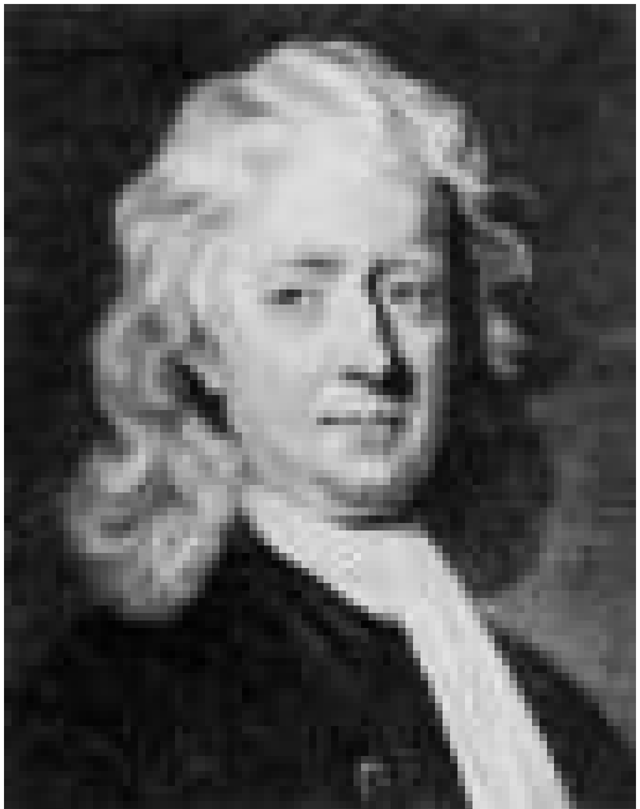


History of Isaac Newton

- 17th Century
  - Shift of progress in math
  - “relative freedom” of thought in Northern Europe



The Life of Newton

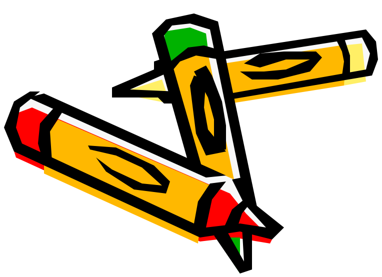


- Born: Christmas day 1642
- Died: 1727
- Raised by grandmother

Newton’s Education

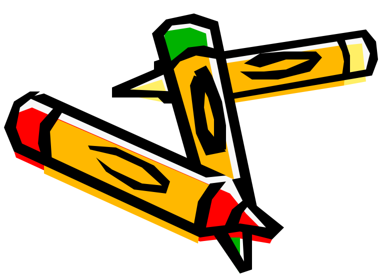
- 1661
- Began at Trinity College of Cambridge University

- 1660
- Charles II became King of England
- Suspicion and hostility towards Cambridge



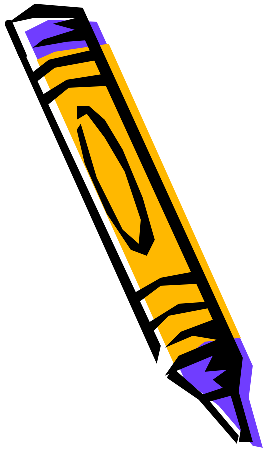
Newton, the young man

- “single minded”
  - Would not eat or sleep over an intriguing problem
- Puritan
  - Book of sins



Newton’s Studies

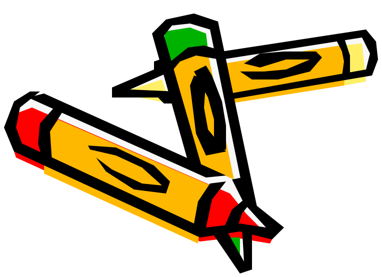
- 1664
  - Promoted to scholar at Trinity
- 1665-1666
  - Plague
  - Newton’s most productive years



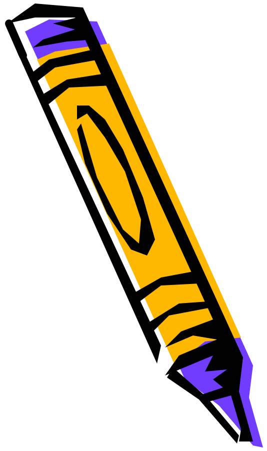
Newton’s Discoveries

- 1665
  - Newton’s “generalized binomial theorem”
  - led to method of fluxions

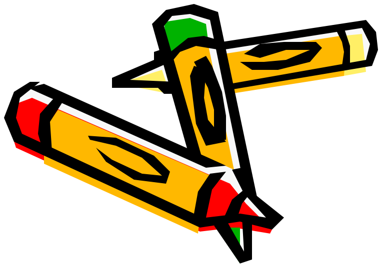
- 1666
  - Inverse method of fluxions
  - Began observations of rotation of planets



Newton's Accomplishments



- 1668
  - Finished master's degree
  - Elected fellow of Trinity College
- 1669
  - Appointed Lucasian chair of mathematics

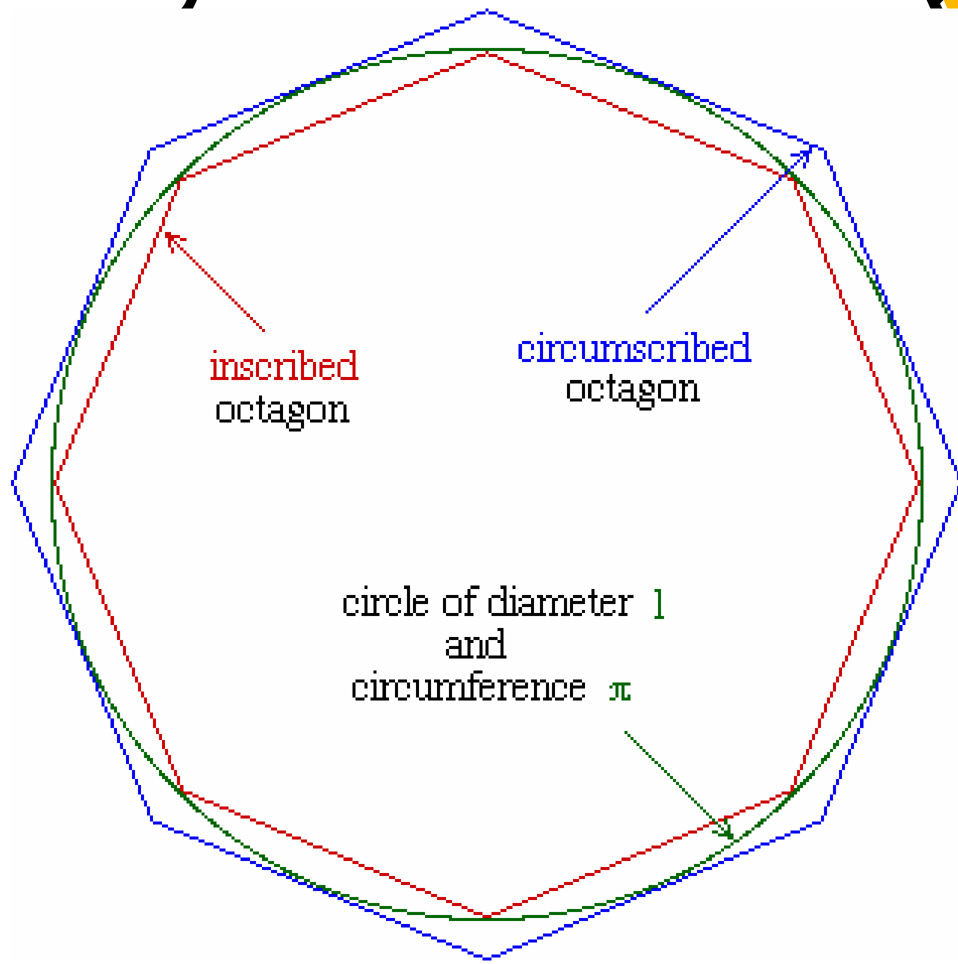


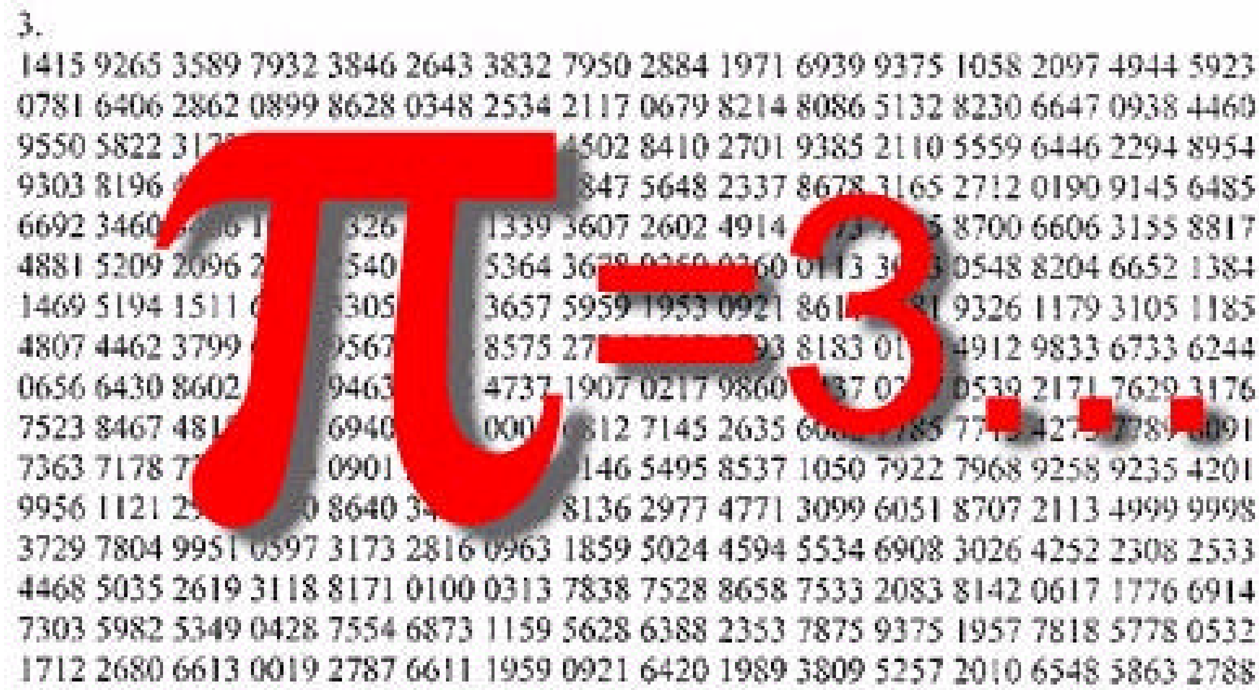
Newton's Accomplishments

- **@ 1704**
  - **Elected President of the Royal Society**
- **1705**
  - **Knighted by Queen Anne**
- **1727**
  - **Buried in Westminster Abbey**

The History of Pi

- Archimedes' classical method
  - Using Polygons with inscribed And Circumscribed circles





– Found Pi between 223/71 and 22/7
  - =3.14

Important Dates of Pi

- 150 AD
  - First notable value for Pi by Caludius Ptolemy of Alexandria
  - Pi = 3 8’30”

  = 377/120

  = 3.1416

- 480 AD
  - TSU Ch'ung-chih from China gave rational approximation
  - Pi = 355/113

    = 3.1415929
- 530 AD
  - Hindu mathematician Aryabhata
  - Pi = 62,832/20,000

    = 3.1416

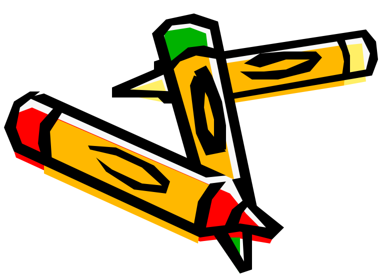
- 1150 AD
  - Bhaskara
  - Pi = 3,927/1250

  Pi = 22/7

  Pi = 754/240

  = 3.1416

- 1429 AD
  - Al- Kashi
  - Astronomer approximated Pi to 16 decimal places
- 1579 AD
  - Francois Viete from France
  - Approximated Pi to 9 decimal places



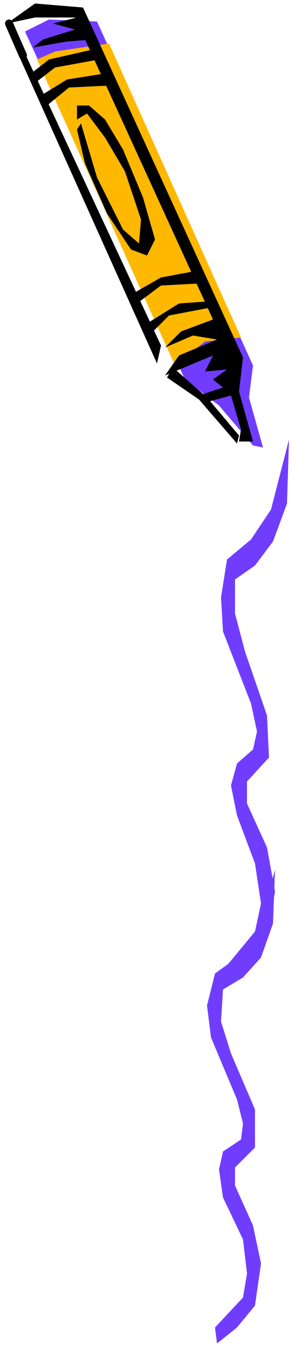
- 1585 AD
  - Adriaen Anthoniszoon
  - Rediscovered Chinese ratio 355/113
  - 377/120> Pi > 333/106
- 1593 AD
  - Adriaen Von Roomen
  - Found Pi to the 15^{th} decimal place by classical method using polygons with

    2^30^{th} sides

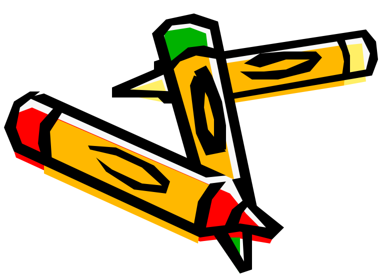
- 1610 AD
  - Ludolph Van Ceulen of the Netherlands
  - Pi ~ 30 decimal places
  - Used polygons with 2^{62} sides
- 1621 AD
  - Willebrord Snell (Dutch)
  - Able to get Ceulen's 35th decimal place by only 2^{30} side polygon

- 1630 AD
  - Grienberger
  - Pi to 39 decimal places
- 1671
  - James Gregory from Scotland obtained infinite series

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots (-1 \le x \le 1)$$



- 1699 AD
  - Abraham Sharp
  - Pi ~ 71 decimal places
- 1706 AD
  - John Machin
  - Pi ~ 100th decimal place



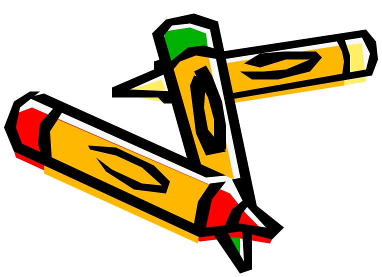
- 1719 AD
  - De Lagny of France
  - Pi ~ 112 decimal places
- 1737 AD
  - William Jones from England
  - First to use Pi symbol for ratio of the circumference to the diameter

- 1767 AD
  - Johan Heinrich Lambert
  - Showed Pi is irrational
- 1794 AD
  - Adrien-Marie Legendre
  - Showed Pi-squared is irrational

- 1841 AD
  - William Rutherford
  - Calculated Pi to 208 places
- 1844 AD
  - Zacharis Dase found Pi correct to 200 places using Gregory Series

$$\frac{p}{2} = \arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{5}\right) + \arctan\left(\frac{1}{8}\right)$$

- 1853 AD
  - Rutherford returns
  - Finds Pi to 400 decimal places
- 1873 AD
  - William Shanks from England
  - Pi to 707 decimal places
- 1882 AD
  - F. Lindeman
  - Shows Pi is transcendental

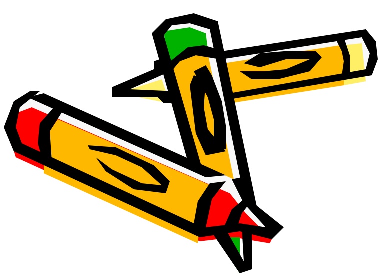


- 1948
  - D.F. Ferguson of England
    - Finds errors with Shanks value of Pi starting with the 528th decimal place
    - Gives correct value to the 710th place
  - J.W. Wrench Jr.
    - Works with Ferguson to find 808th place for Pi

Used Machin's formula

$$\frac{p}{4} = 3\arctan\left(\frac{1}{4}\right) + \arctan\left(\frac{1}{20}\right) + \arctan\left(\frac{1}{1985}\right)$$

- 1949 AD
  - Electronic computer – The ENIAC
    - Compute Pi to the 2,037th decimal places
- 1959 AD
  - Fancois Genuys from Paris
  - Compute Pi to 16,167 decimal places with IBM 704



- 1961 AD
  - Wrench and Shanks of Washington D.C.
  - compute Pi to 100,265th

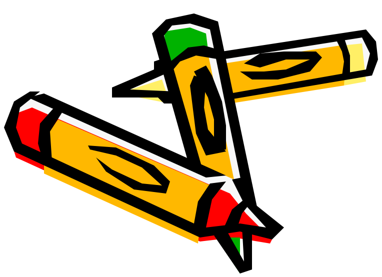
  using IBM 7090
- 1966 AD
  - M. Jean Guilloud and co-workers
  - attained approximation for Pi

  to 250,000 decimal places on a STRETCH computer

- 1967 AD
  - M. Jean Guilloud and coworkers
  - found Pi to the 500,000 places on a CDC 6600

- 1973
  - M. Jean Guilloud and coworkers found Pi to 1 millionth place on CDC 7600

- 1981 AD
  - Kazunori Miyoshi and Kazuhika Nakayma of the University of Tsukuba
  - Pi to 2 million and 38 decimal places in 137.30 hours on a FACOM M-200 computer

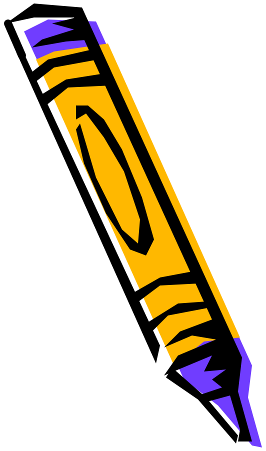


- 1986 AD
  - DH Bailey of NASA Ames Research Center ran a Cray-2 supercomputer for 28 hours
    - Got Pi to 29,360,000 decimal places
  - Yasamasa Kanada from University of Tokyo
    - Used NEC SX-2 super computer to compute Pi to 134,217,700 decimal places

Purpose to Continue to Compute Pi

- See if digits of Pi start to repeat
  - Possible normalcy of Pi
- Valuable in computer science for designing programs

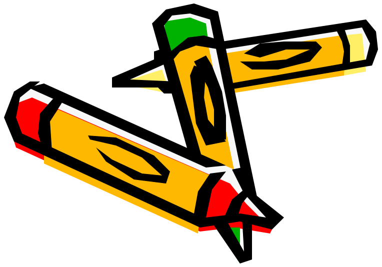
Information Already known



$$\left(x-\frac{1}{2}\right)^2+\left(y-0\right)^2=\frac{1}{2}^{2}$$

or

$$x^2-x+{}^{1}\!/_{4}+y^2={}^{1}\!/_{4}$$



Solve for “y”

$$y = x^{1/2}(1-x)^{1/2}$$

$$= x^{1/2}\left(1-\frac{1}{2}x-\frac{1}{8}x^{2}-\frac{1}{16}x^{3}-\frac{5}{128}x^{4}-\frac{7}{256}x^{5}-\ldots\right)$$

$$= x^{1/2}-\frac{1}{2}x^{3/2}-\frac{1}{8}x^{5/2}-\frac{1}{16}x^{7/2}-\frac{5}{128}x^{9/2}-\frac{7}{256}x^{11/12}-\ldots$$

Area (ABD) by fluxion

$$\frac{2}{3}x^{3/2} - \frac{1}{2}\left(\frac{2}{5}x^{5/2}\right) - \frac{1}{8}\left(\frac{2}{7}x^{7/2}\right) - \frac{1}{16}\left(\frac{2}{9}x^{9/2}\right) - \ldots$$

$$= \frac{2}{3}x^{3/2} - \frac{1}{5}x^{5/2} - \frac{1}{28}x^{7/2} - \frac{1}{72}x^{9/2} - \frac{5}{704}x^{11/2} - \ldots$$

$$\left(\frac{1}{4}\right)^{3/2} = \left(\sqrt{\frac{1}{4}}\right)^{3} = \frac{1}{8}\underline{\qquad},\ \underline{\qquad}\left(\frac{1}{4}\right)^{5/2} = \left(\sqrt{\frac{1}{4}}\right)^{5} = \frac{1}{32}\underline{\qquad}\cdots$$

$$\frac{1}{12} - \frac{1}{160} - \frac{1}{3584} - \frac{5}{1441792} \ldots. - \frac{429}{163208757248} = .07677310678$$

Area (ABD) by geometry

$$\overline{BD} = \sqrt{\left(\frac{1}{2}\right)^2 - \left(\frac{1}{4}\right)^2} = \sqrt{\frac{3}{16}} = \frac{\sqrt{3}}{4}$$

$$Area(\Delta DBC) = \frac{1}{2}\left(\overline{BC}\right)x\left(\overline{BD}\right) = \frac{1}{2}\left(\frac{1}{4}\right)\left(\frac{\sqrt{3}}{4}\right) = \frac{\sqrt{3}}{32}$$

$$Area(\sec tor) = \frac{1}{3} Area(semicircle)$$

$$= \frac{1}{3}\left(\frac{1}{2} \cdot p \cdot r^2\right)$$

$$= \frac{1}{3}\left[\frac{1}{2} p \left(\frac{1}{2}\right)^2\right]$$

$$= \frac{p}{24}$$

$$Area(ABD) = Area(\sec tor) - Area(\Delta DBC)$$

$$= \frac{p}{24} - \frac{\sqrt{3}}{32}$$

$$p \approx 24\left(.07677310678 + \frac{\sqrt{3}}{32} \right) = 3.141592668...$$