The next corpus of extant mathematical documents appeared after Egypt came under Persian rule (525–404 BC and 343–332 BC) and then became part of the Hellenistic world (332 BC), finally ending as a province of the Roman empire (30 BC). The texts were recorded in the demotic script. It was during this period that Egyptian mathematics came under the increasing influence of Mesopotamia and Greece. We conclude our discussion of Egyptian mathematics with an example found in Parker (1972, pp. 35–37) taken from a text dated to around the fourth century BC. Similar problems exist in Mesopotamian mathematics and later in Chinese mathematics. Whether this indicates transmissions farther afield is an intriguing question for which no definitive evidence exists.

EXAMPLE 3.17 The foot of a pole [length] 10 cubits is moved outward so that its top is [resting] 8 cubits vertically. By how much has the top of the pole been lowered?

EGYPTIAN EXPLANATION	MODERN EXPLANATION
1. You should reckon 10, 10 times: result 100.	Let a = length of the pole.
2. You should reckon 8, 8 times: result 64.	Let c = height of the pole once it is moved out.
3. Take it from 100: result 36.	Let b = the distance that the foot of the pole has moved out.
4. Reduce to its square root: result 6.	$c^2 = a^2 - b^2 = 10^2 - 8^2 = 36 \rightarrow c = 6$ $a - c = 4$.
5. Take it from 10: remainder 4.	

You shall say: "Four cubits is the answer."

It would seem that the Pythagorean theorem was applied in arriving at this solution.

Egyptian Mathematics: A General Assessment

Egyptian mathematics has been discussed here in more detail than in some of the past textbooks on the early history of mathematics.The treatment of Egyptian mathematics in many of these texts tends to be rather lopsided: Egyptian numeration is overemphasized, and consequently the rest of the mathematics receives less attention than it should. Where comparisons are made with contemporary or later mathematical traditions, the quality of Mesopotamian mathematics is stressed, but both the Egyptian and Mesopotamian contributions are judged to be meager, or—more charitably—seen merely as a prelude to the "Greek miracle." However, it should be pointed out that the amount of space devoted to Egyptian mathematics by some of the recent texts such as Burton (2005), Cooke (1997), Katz (1998), and Suzuki (2001) has increased considerably over the years.

Over several years, both on the Internet and in academic journals, there has been a lively debate on the historical relationship of Greek to Egyptian science in general, and mathematics in particular. The start of this debate in recent years may be traced to Martin Bernal's (1987, 1992, 1994) claim that Classical Greek culture was significantly influenced by ancient Egyptian civilization, partly through Egyptian colonization of parts of Greece. And this influence was also reflected in the debt that Greek science owed to its Egyptian counterpart. A debate that began between Bernal (1992, 1994) and Palter (1993) has spilled over to journals and popular magazines. Interesting interventions by Victor Katz and Beatrice Lumpkin have been published in a newsletter in July 1995. The discussion that follows has been influenced by their contributions.

The arguments marshaled by Katz relate to two claims made by Bernal and rejected by Palter:

1. There were scientific elements in Egyptian medicine, mathematics, and astronomy long before there was any Greek science at all.

2. Egyptian medicine, mathematics, and astronomy significantly influenced the development of corresponding Greek disciplines.

In relation to mathematics, we know from the evidence contained in this chapter that the Egyptians certainly knew how to solve various kinds of problems, from solving what we would now describe as linear equations to calculating the volumes and areas of different geometrical objects, including possibly the surface of a hemisphere. Lumpkin (2000) has summarized the nature and extent of the Egyptian contribution to science (and in particular mathematics long before the appearance of Greek science). What we are uncertain about is how the Egyptians discovered the methods they used in solving the problems. Presumably, at the very least, there was some "scientific" underpinning to their methods, although not necessarily based on reasoning from explicit axioms. On the other issue of whether

Egyptian mathematics influenced Greek mathematics, it may be possible to give a more definitive "yes." We have pointed out a similarity between the mathematical thinking of the Egyptians and the Greeks when it came to number theory. After all, many of the ancient Greek sources acknowledge this influence. As Katz (1995, pp. 10–11) stated:

> Not only is Pythagoras supposed to have studied in Egypt, but so is Thales, the supposed father of Greek geometry. Also, Oenopides. Herodotus, Heron of Alexandria, Diodorus Siculus, Strabo, Socrates (through Plato) and Aristotle—all say that geometry was first invented by the Egyptians and then passed on to the Greeks. The question always seems to be, in this regard, what we mean by geometry. If, by geometry, we mean an axiomatic treatment with theorems and proofs in the style of Euclid, then it is clear that this was a Greek invention. But mathematicians have always known that, in general, one does not discover theorems by the axiomatic method. One discovers theorems by experiment, by trial and error, by induction, etc. Only after the discovery is there a search for a rigorous "proof."

Srinivas Ramanujan's mathematics, discussed in the preface to the first edition of this book, is a good example of the discovery and the "proof" being undertaken by different persons and at different times. And it would seem clear that when the Greeks declare that the Egyptians invented (or discovered) geometry, it is the results that they have in mind and not necessarily the method of proof. Conceding that the Greeks learned various geometrical results from the Egyptians takes nothing away from Greek creativity. They were simply doing what mathematicians have always done, building on the results of their predecessors.

In chapter 1, we examined the issue of transmission between different mathematical traditions, concentrating mainly on the connections that arose during and after the Dark Ages in Europe. Some of the most promising recent research, notably that of Friberg (2005, 2007b), relates to discovering the three-way links that have been neglected so far: those between the Egyptian, Mesopotamian, and Greek mathematics. Past researchers have usually considered Egyptian and Mesopotamian mathematics as completely independent mathematical traditions despite their proximity to each other in time and space. Their differences, notably in the development of a decimal, nonpositional number system in Egypt compared with a sexagesimal positional number system in Mesopotamia, have been emphasized. Further, the persistence of a belief in the "Greek miracle" in some guise or other, alluded to in chapter 1, has underpinned the view of Greek mathematics as being unique and independent of the earlier mathematical traditions of Egypt and Mesopotamia. However, today there is far greater attention paid to the methodology of establishing transmission as well as a growing recognition that the cultural context in which a mathematical document arises is a crucial consideration as to its interpretation. In chapter 5, in the assessment of Egyptian and Mesopotamian mathematics, the linkages between these two mathematical traditions as well as the traces of these traditions in Greek mathematics will be examined in greater detail.

We have said little about the later phase of Egyptian mathematics, when Alexandria became the center of mathematical activity. It was the creative synthesis of Classical Greek mathematics, with its strong geometric and deductive tradition, and the algebraic and empirical traditions of Egypt and Mesopotamia that produced some of the greatest mathematics and astronomy of antiquity, best exemplified in the works of Archimedes, Ptolemy, Diophantus, Pappus, and Heron. We shall not take up the story of Hellenistic mathematics, which has been extensively explored in general histories, such as those by Boyer (1968), Eves (1983), Katz (1998), and Kline (1972), as well as in specialized works on Greek mathematics by Cuomo (2001), Fowler (1987), Heath (1921), van der Waerden (1961), and others.

Notes

1. Diodorus has been criticized as exhibiting "none of the critical faculties of the historian (but) merely setting down a number of unconnected details." But his English translator, Oldfather (1989), reminds us that of all the forty volumes of his *Universal History*, the first volume on Egypt (published in 1960) is the "fullest literary account of the history and customs of that country after Herodotus."

2. Davidson (1987, pp. 1–2), writing about public reactions to a television series that he presented on the history of the Africans, points out that what a number of viewers in Europe and North America found particularly difficult to accept was the "black" origins of the ancient Egyptians: "To affirm this, of course, is to offend nearly all established historiographical orthodoxy."

3. *Ujamaa* is a Swahili word meaning "brotherhood" that was used to describe a Tanzanian government initiative in the late 1960s and early 1970s to encourage scattered rural homesteads to form villages, which would then serve both as pools of labor for communal activities and as units for meeting social needs in health, education, communication, and water supply.

4. In fairness, it should be added that this is perceived as a fanciful story by some scholars of predynastic Egypt. For a useful review of evidence, see Bard (1994, pp. 265–88).

5. The New York fragments consist only of a small table and the early section of the problems (nos. 1–6), which helped to complete the whole text.

6. For the contents of these deciphered fragments, see Imhausen and Ritter (2004) and Imhausen (2006).

7. Friberg identfies a codex of six papyrus leaves and a small corpus of "non-Euclidean" Greek mathematical texts.

8. For further details, see Imhausen (2007, pp. 10–11).

9. One cubit is equal to approximately 52.5 cm.

10. See Fischer-Elfert (1986, pp. 118–57) for a discussion of the different interpretations.

11. There are different interpretations of the numbers shown on the mace head. Petrie, an early Egyptologist, suggested that the mace head depicted scenes of a political marriage of Narmer to a princess from the north at which he received tributes from different people. Others have interpreted it as recording the spoils of war of Narmer after his conquest of the north. A third interpretation suggests a census of the male population and their livestock taken during his reign.

12. For further details, see Resnikoff amd Wells (1984, p. 23).

13. A highly cursive form of hieratic known as "abnormal hieratic," derived from the script of Upper Egyptian administrative documents, was used primarily for legal texts, land leases, letters, and other texts. This type of writing was superseded by demotic—a Lower Egyptian scribal tradition—and became the standard administrative script throughout a reunified Egypt.

14. However, Bruckheimer and Salamon (1977) have argued that in a number of cases the selection criteria put forward by Gillings are inappropriate. A more recent critique of Gillings's procedure is found in Abdulaziz (2008).

15. Apart from table texts for computing unit fractions, other tables to convert different measuring units (including measures of volume) and tables used as aids to calculation have been discovered.

16. The figure given in the text is 2,301, which is incorrect.

17. The following were the units of measurement of length in ancient Egypt: 1 *chet* = 100 cubits; 1 cubit = 7 palms; 1 palm = 4 digits. In terms of modern measurement, 1

cubit ≈ 52.5 cm; 1 palm ≈ 7.5 cm, and 1 digit ≈ 19 mm. A "trigonometric" interpretation of the *seked* has been offered on the basis of its being a ratio of width to height, which is therefore equivalent to the cotangent of the relevant angle. Since there is no notion of measurement of an angle in Egyptian mathematics (see endnote 17, chapter 1), such an interpretation would seem somewhat far-fetched.

18. It may be argued that the explanations given are contrived, unconvincing, and lacking in any real evidence to back them. To the author, however, their merit lies in the fact that there is an underlying materialistic basis to these conjectures.

19. The Heronian mean of two positive numbers x and y is given by $(1/3)(x + y + \sqrt{xy})$. For further details, see Bullen (2003).