

MA 765 Homework 3

Due Friday, February 23

Let k be an algebraically closed field.

1. Let $K \subseteq k(t)$ be a field of transcendence degree 1 over k . Show that K is isomorphic to $k(t)$. (Hint: show that K is finitely generated over k , and then use the Euclidean algorithm.)
2. Conclude that, if C is an irreducible smooth projective curve and $\varphi : \mathbb{P}^1 \rightarrow C$ is a non-constant morphism, then C is isomorphic to \mathbb{P}^1 .
3. Show that, for any irreducible smooth projective curve C , there exists a surjective morphism $\psi : C \rightarrow \mathbb{P}^1$.