

MA 361 Homework 3
Due Friday, September 18

1. Prove that, if G is a group with the property that the square of every element is the identity, then G is abelian.
2. Prove that, in any group, an element and its inverse have the same order.
3. Suppose that a is an element of a group G and $a^6 = e$, where e is the identity. What are the possibilities for the order of a ? Provide reasons for your answers.
4. Find groups that contain elements a and b such that $|a| = |b| = 2$ and
 - (a) $|ab| = 3$,
 - (b) $|ab| = 4$,
 - (c) $|ab| = 5$.
5. If H and K are subgroups of a group G , show that $H \cap K$ is a subgroup of G .
6. Provide a counterexample to the following statement: if H and K are subgroups of a group G , then $H \cup K$ is a subgroup of G .