

## MA 565 Homework 10

Due Friday, November 6

Axler 5.C # 6, 16

1. Prove that two  $2 \times 2$  matrices which are not scalar matrices are similar if and only if they have the same characteristic polynomial.
2. Prove that two  $3 \times 3$  matrices are similar if and only if they have the same characteristic and minimal polynomials. Give an explicit counterexample to this assertion for  $4 \times 4$  matrices.
3. Find all similarity classes of  $6 \times 6$  matrices over  $\mathbb{Q}$  with minimal polynomial  $(x + 2)^2(x - 1)$ .
4. Find all similarity classes of  $6 \times 6$  matrices over  $\mathbb{C}$  with characteristic polynomial  $(x^4 - 1)(x^2 - 1)$ .
5. Find all similarity classes of  $3 \times 3$  matrices over  $\mathbb{F}_2$  satisfying  $A^6 = \text{Id}$ .
6. Let  $V$  be a finite dimensional vector space over  $\mathbb{Q}$  and suppose that  $T$  is a nonsingular linear transformation of  $V$  such that  $T^{-1} = T^2 + T$ . Prove that the dimension of  $V$  is divisible by 3. If the dimension of  $V$  is precisely 3, prove that all such linear transformations  $T$  are similar.