MA 565 Homework 14

Due Friday, December 11

Axler 6.C # 7, 8, 14

Axler 10.B # 12

1. (a) Show that $\mathbb{R}^{\oplus\infty}$ is an inner product space under the inner product

$$\langle (x_1, x_2, \ldots), (y_1, y_2, \ldots) \rangle = \sum_{i=1}^{\infty} x_i y_i.$$

   (b) Use this example to show that the Riesz representation theorem does not hold for infinite dimensional inner product spaces.

2. Let V and W be inner product spaces. A linear map $T : V \to W$ is *bounded* if there exists a constant $K \geq 0$ such that $\|T(v)\| \leq K\|v\|$ for all $v \in V$.

   (a) Show that, if V and W are finite dimensional, then any linear map $T : V \to W$ is bounded.

   (b) Find an example of a linear map between inner product spaces that is not bounded.

   (c) A function $T : V \to W$ is *continuous* if, for any $\epsilon > 0$, there exists a $\delta > 0$ such that, whenever $\|v - u\| < \delta$ for some $u, v \in V$, we have $\|T(v) - T(u)\| < \epsilon$. Show that a linear map $T : V \to W$ is continuous if and only if it is bounded.