MA 565 Homework 4
Due Friday, September 25

Axler Chapter 3E # 2,4,5,6

1. Given linear maps $T : U \to V$ and $S : W \to V$, the *fiber product* $U \times_V W$ is defined to be

$$U \times_V W = \{(u, w) \in U \times W | T(u) = S(w)\}.$$

   (a) Show that $U \times_0 W$ is equal to $U \times W$.

   (b) Show that, if $S : W \to W$ is the identity, then $U \times_W W$ is equal to the graph of T.

   (c) Let X be a vector space and suppose that there exist linear maps $T' : X \to U$, $S' : X \to W$ making the following diagram commute:

$$\begin{array}{ccc} X & \xrightarrow{T'} & U \\ \downarrow{\scriptstyle S'} & & \downarrow{\scriptstyle T} \\ W & \xrightarrow{S} & V. \end{array}$$

   Show that there exists a unique linear map $\varphi : X \to U \times_V W$ making the following diagram commute: