

## MA 565 Homework 5

Due Friday, October 2

Axler Chapter 3E # 10, 13, 14, 15, 18, 20

1. Prove that

$$0 \rightarrow U \rightarrow V \rightarrow W \rightarrow 0$$

is an exact sequence if and only if  $W \cong V/U$ .

2. Prove that the sequence

$$0 \longrightarrow k[x] \xrightarrow{\cdot x} k[x] \xrightarrow{\text{ev}_0} k \longrightarrow 0$$

is exact, where the first map is multiplication by  $x$ , and the second map is evaluation at zero.

3. Suppose that

$$0 \longrightarrow V_1 \xrightarrow{\varphi_1} V_2 \xrightarrow{\varphi_2} \cdots \xrightarrow{\varphi_{n-1}} V_n \longrightarrow 0$$

is an exact sequence of finite dimensional vector spaces. Show that

$$\sum_{i=1}^n (-1)^i \dim(V_i) = 0.$$