MA 565 Homework 9
Due Friday, October 30

Axler, Chapter 5A # 5,6,8,12,15,20,21,25,29

1. Let U and V be vector spaces, and suppose that, for every vector space W, there is an isomorphism

$$\varphi_W : \operatorname{Hom}(U, W) \to \operatorname{Hom}(V, W).$$

Suppose moreover that, for every linear map of vector spaces

$$T : W_1 \to W_2,$$

the following diagram commutes:

$$
\begin{array}{ccc}
\operatorname{Hom}(U, W_1) & \xrightarrow{\ f \mapsto T \circ f\ } & \operatorname{Hom}(U, W_2) \\
\downarrow{\scriptstyle \varphi_{W_1}} & & \downarrow{\scriptstyle \varphi_{W_2}} \\
\operatorname{Hom}(V, W_1) & \xrightarrow{\ g \mapsto T \circ g\ } & \operatorname{Hom}(V, W_2)
\end{array}
$$

Prove that U is isomorphic to V.