MA 665 EXERCISES 12

(1) Let k be an algebraically closed field, I an ideal in $R = k[x_1, \ldots, x_n]$, and $X = V(I)$. Show that there is a 1-to-1 correspondence between closed subsets of X and radical ideals in R/I. Show that, under this correspondence, maximal ideals in R/I correspond to points in X.

(2) Let $R \subseteq S \subseteq T$ be rings. Show:
  (a) If S is finitely generated as a module over R and T is finitely generated as a module over S, then T is finitely generated as a module over R.
  (b) If S is finitely generated as a ring over R and T is finitely generated as a ring over S, then T is finitely generated as a ring over R.

(3) Let k be a field.
  (a) Show that the set of elements of $k(x)$ that are integral over $k[x]$ is $k[x]$ itself.
  (b) Show that there is no nonzero element $F \in k[x]$ such that, for every $z \in k(x)$, there exists an n such that $F^n z$ is integral over $k[x]$.