MA 665 EXERCISES 13

(1) Let R be a commutative ring with unit, and $x \in R$ an element that is neither a unit nor a zero divisor. Prove that the set of associated primes of $R/(x^n)$ is equal to the set of associated primes of $R/(x)$ for all $n \geq 1$.

(2) Let M be a finitely generated R-module. The *support* of M is the set of prime ideals $P \subset R$ such that $M_P \neq 0$.
   (a) Prove that every associated prime of M is contained in the support of M.
   (b) Show that if P is a *minimal* element of the support of M, then P is an associated prime of M.

(3) Let $S \subset R$ be a multiplicative set, M an R-module, and $N, N' \subseteq M$ submodules of M. Prove that $(N \cap N')_S = N_S \cap N'_S$.