MA 665 EXERCISES 6

(1) Show that the ring homomorphism $\mathbb{Z} \hookrightarrow \mathbb{Q}$ is both a monomorphism and an epimorphism in the category of commutative rings with unit, though it is not an isomorphism.

(2) If $\mathcal{G}$ and $\mathcal{H}$ are groups regarded as categories with one object, characterize functors from $\mathcal{G}$ to $\mathcal{H}$.

(3) Find an example of a ring R and a module M such that $\mathrm{Hom}_R(M, -)$ is not full. Similarly, find an example such that $\mathrm{Hom}_R(M, -)$ is not faithful.