CHIP FIRING EXERCISES 3

(1) Let G be a graph and v a vertex of G. Show that an acyclic orientation $\mathcal{O}$ is v-connected if and only if $D_{\mathcal{O}}$ is v-reduced. Give an example of a graph G, a vertex v, and a v-connected orientation $\mathcal{O}$ such that $D_{\mathcal{O}}$ is not v-reduced.

(2) Let G be a graph of genus g. Show that a divisor D on G of degree $g-1$ is equivalent to an effective divisor if and only if $K_G - D$ is equivalent to an effective divisor.

(3) Let G be the complete graph on n vertices and let $D = \sum_{i=1}^{n} a_i v_i$ be a divisor on G. Reorder the vertices so that $a_1 \leq a_2 \leq \cdots \leq a_n$. Prove that D is a break divisor if and only if $\sum_{i=1}^{k} a_i \geq \binom{k-1}{2}$ for all k, with equality when $k = n$.