CHIP FIRING EXERCISES 4

(1) Without using Riemann-Roch, prove the following:
  (a) A divisor of nonnegative degree on a tree has rank equal to its degree.
  (b) A divisor of positive degree on a cycle has rank one less than its degree.

(2) Find two graphs, each of genus 3, one of which has a divisor of degree 2 and rank 1, and the other of which does not.

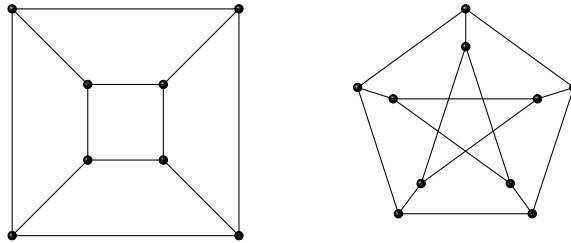(3) On each of the two graphs depicted below, find a rank one divisor of minimal degree.

FIGURE 1