# Basic Rules of Differentiation

The <u>derivative</u> of a function $f(x)$ at $x$ is the rate at which $f(x)$ is changing at $x$.

It is also the slope of the tangent line to the graph of $y=f(x)$ at the point $(x, f(x))$.

The formal definition is

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}.$$

Last time, we considered the function $f(x)=x^3$, and we computed $f'(1)$.

<u>Ex</u>: Today, let's compute $f'(x)$.

$$f(x) = x^2.$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \lim_{h\to 0} \frac{2xh + h^2}{h} = \lim_{h\to 0} 2x + h = 2x$$

$$\boxed{\frac{d}{dx}\left[x^2\right] = 2x}$$

Ex: Let $f(x) = x^n$ where $n$ is a nonnegative integer.

Compute $f'(x)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^n - x^n}{h} = \lim_{h\to 0} \frac{\overbrace{(x+h)\cdot(x+h)\cdots(x+h)}^{n \text{ times}} - x^n}{h}$$

$$= \lim_{h\to 0} \frac{x^n + n\cdot h\cdot x^{n-1} + h^2\cdot[\text{stuff}] - x^n}{h}$$

$$= \lim_{h\to 0} \frac{n\cdot h\cdot x^{n-1} + h^2\cdot[\text{stuff}]}{h}$$

$$= \lim_{h\to 0} n\cdot x^{n-1} + h\cdot[\text{stuff}] = nx^{n-1}$$

**Power Rule**: If $n$ is a nonnegative integer, then $\frac{d}{dx}\left[x^n\right] = nx^{n-1}$

Ex: What is the derivative of $f(x) = x^{71}$?

Use the power rule with $n = 71$.

$$\frac{d}{dx}\left[x^{71}\right] = 71 \cdot x^{71-1} = 71 \cdot x^{70}$$

**Constant Multiple Rule:** If $c$ is a constant, then $\frac{d}{dx}\left[c \cdot f(x)\right] = c \cdot \frac{d}{dx}\left[f(x)\right]$

Why? $\frac{d}{dx}\left[c \cdot f(x)\right] = \lim_{h \to 0} \frac{c \cdot f(x+h) - c \cdot f(x)}{h}$

$$= \lim_{h \to 0} c \cdot \frac{f(x+h) - f(x)}{h} = c \cdot \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= c \cdot \frac{d}{dx}\left[f(x)\right]$$

**Sum Rule:**

$$\frac{d}{dx}\left[f(x) + g(x)\right] = \frac{d}{dx}\left[f(x)\right] + \frac{d}{dx}\left[g(x)\right]$$

Why? $\frac{d}{dx}\left[f(x) + g(x)\right] = \lim_{h \to 0} \frac{f(x+h) + g(x+h) - [f(x) + g(x)}{h}$

$$= \lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h} + \frac{g(x+h)-g(x)}{h}\right]$$

$$= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} + \lim_{h\to 0}\frac{g(x+h)-g(x)}{h}$$

$$= \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)].$$

**Ex:** Compute $\frac{d}{dx}[x^3+7x^2-5x+12]$

SUM RULE
$$= \frac{d}{dx}[x^3] + \frac{d}{dx}[7x^2] + \frac{d}{dx}[-5x] + \frac{d}{dx}[12]$$

CONSTANT MULTIPLE RULE
$$= \frac{d}{dx}[x^3] + 7\frac{d}{dx}[x^2] - 5\frac{d}{dx}[x] + 12\frac{d}{dx}[1]$$

POWER RULE
$$= 3x^2 + 7\cdot 2x - 5\cdot 1 + 12\cdot 0$$

derivative of a constant is 0

$$= 3x^2 + 14x - 5$$

**Ex:** Compute $\frac{d}{dx}[x^{100}+31x^{17}-3x+5]$

$$= \frac{d}{dx}[x^{100}] + \frac{d}{dx}[31x^{17}] + \frac{d}{dx}[-3x] + \frac{d}{dx}[5]$$

$= \frac{d}{dx}[x^{100}] + 31\frac{d}{dx}[x^{17}] - 3\frac{d}{dx}[x] + \frac{d}{dx}[5]$

$= 100 \cdot x^{99} + 31 \cdot 17x^{16} - 3 \cdot 1 + 0$

$= 100x^{99} + 527x^{16} - 3$

WARNING! Derivative of a product is NOT the product of the derivatives!

Ex: $\frac{d}{dx}[x^2] = 2x$ $\qquad$ $\frac{d}{dx}[x] = 1$

$\frac{d}{dx}[x \cdot x] \neq \frac{d}{dx}[x] \cdot \frac{d}{dx}[x] = 1$

Next time, we'll see how to compute the derivative of a product of 2 functions.