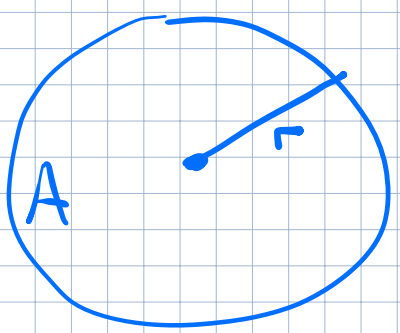
Related Rates

Today is the last day to fill out the course survey (on Canvas under Quizzes)

Ex: The radius of a circular puddle is increasing at a rate of 3m/hour. When the radius of the puddle is 2m, how quickly is the area of the puddle increasing?

① Draw a picture



r = radius

A = area

② What are we given? What are we trying to find?

Given: $\frac{dr}{dt} = 3$ m/hour

Want: Find $\frac{dA}{dt}$ when $r = 2$ m.

③ Write down an expression that relates the relevant functions.

$$A = \pi \cdot r^2 .$$

④ Take the derivative of both sides.
(Remember to use the chain rule.)

$$\frac{d}{dt}[A] = \frac{d}{dt}[\pi r^2]$$

$$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$$

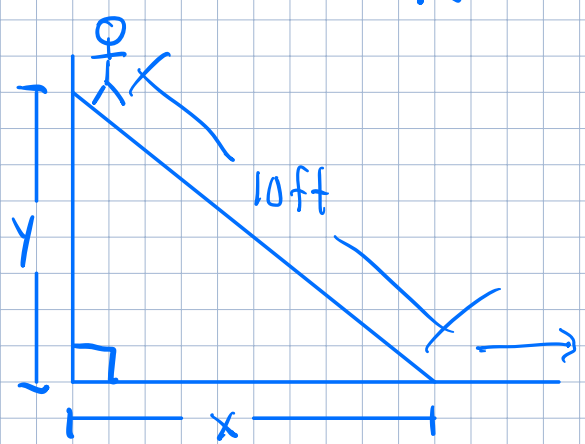
CHAIN RULE

⑤ Plug in

$$\frac{dA}{dt} = 2\pi \cdot 2 \cdot 3 = 12\pi \quad m^2/hour.$$

Ex: A 10ft ladder is propped against a wall. If the base of the ladder slips away from the wall at a rate of 1ft/s, how quickly is the top of the ladder falling when the top of the ladder is 8ft from the floor?

① Draw a picture and label all the important variables



② What are you given? What are you trying to find?

Given: $\frac{dx}{dt} = 1$ ft/s

Want to find:

$\frac{dy}{dt}$ when $y = 8$ ft

③ Write down an expression relating the variables

$x^2 + y^2 = 10^2$ (Pythagorean Theorem)

④ Take the derivative of both sides

$$\frac{d}{dt}[x^2 + y^2] = \frac{d}{dt}[100]$$

$$2x \cdot \frac{dx}{dt} + 2y \frac{dy}{dt} = 0.$$

⑤ Plug in

$\frac{dx}{dt} = 1$, $y = 8$ to find $\frac{dy}{dt}$

$x^2 + y^2 = 100$

$x^2 + 8^2 = 100$

$x^2 + 64 = 100$

$x^2 = 36$

$x = 6$

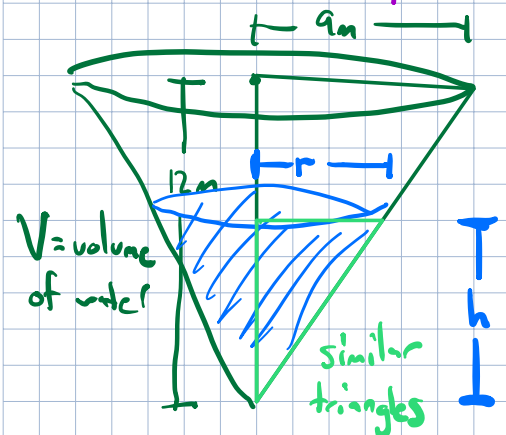
$$2 \cdot 6 \cdot 1 + 2 \cdot 8 \cdot \frac{dy}{dt} = 0$$

$$12 + 16 \cdot \frac{dy}{dt} = 0 \qquad 16\frac{dy}{dt} = -12$$

$$\frac{dy}{dt} = \frac{-12}{16} = -\frac{3}{4} \text{ ft/s}$$

Ex: A water tank is shaped like an inverted cone with height 12 m and base radius 9 m. Water is pumped in at a rate of 2 L/hour. At what rate is the depth of the water increasing when the depth is 4 m?

(1) Draw a picture and label all variables



(2) What are we given? What are we trying to find?

Given: $\frac{dV}{dt} = 2$ L/hour

Want to find: $\frac{dh}{dt}$ when $h = 4$ m.

(3) Write down an expression relating the variables.

$$V = \frac{1}{3}\pi h r^2$$

Formula for the volume of a cone

It would be nice to simplify this expression to eliminate the variable r.

$$\frac{r}{h} = \frac{9}{12} = \frac{3}{4} \qquad r = \frac{3}{4}h$$

$$V = \frac{1}{3}\pi h r^2 = \frac{1}{3}\pi h\left(\frac{3}{4}h\right)^2$$

$$V = \frac{3}{16}\pi h^3$$

(4) Take the derivative of both sides

$$\frac{dV}{dt} = \frac{d}{dt}\left[\frac{3}{16}\pi h^3\right]$$

$$\frac{dV}{dt} = \frac{3}{16}\pi \cdot 3h^2 \cdot \frac{dh}{dt} = \frac{9}{16}\pi h^2 \frac{dh}{dt}$$

(5) Plug in

$$2 = \frac{9}{16}\pi \cdot \cancel{4^2} \cdot \frac{dh}{dt}$$

$$2 = 9\pi \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{2}{9\pi} \quad m/hour$$