MA 137 - Lecture 2

First Exam IN CLASS Feb. 2nd

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Office Hours: MW 12:30 - 2 Zoom

NOT TODAY

Also: TA Office Hours

Mathskellar - in the basement of CB

M-F 9-4

The Study

Exponential Functions

An exponential function is a function of the form

$f(x) = a^x$, where a is a constant.

Ex: $f(x) = 2^x$. Ex: $f(x) = \left(\frac{1}{3}\right)^x$.

What does that mean?

If x is a positive whole number, then

$$a^x = \underbrace{a \cdot a \cdot a \cdots a}_{x \text{ times}}$$

Ex: $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$= 32$.

Properties of Exponentials:

(1) $a^{x+y} = a^x \cdot a^y$

Why? $a^{x+y} = \underbrace{a \cdot a \cdot a \cdots a}_{x+y \text{ times}} = \underbrace{(a \cdot a \cdots a)}_{x \text{ times}} \cdot \underbrace{(a \cdot a \cdots a)}_{y \text{ times}}$

$= a^x \cdot a^y$

(2) $a^0 = 1$.

<u>Why?</u> $\cancel{a^0} = a^{0+0} \overset{(1)}{=} a^0 \cdot \cancel{a^0}$ Divide both sides by a^0

$1 = a^0$.

(3) $a^{-x} = \frac{1}{a^x}$.

<u>Why?</u> $1 \overset{(2)}{=} a^0 = a^{x+(-x)} \overset{(1)}{=} a^x \cdot a^{-x}$ Divide both sides by a^x.

$\frac{1}{a^x} = a^{-x}$.

(4) $(a \cdot b)^x = a^x \cdot b^x$

<u>Why?</u> $(a\cdot b)^x = \underbrace{(a\cdot b)\cdot(a\cdot b)\cdot \cdots (a\cdot b)}_{x \text{ times}}$

$= \underbrace{(a\cdot a \cdots a)}_{x \text{ times}}\underbrace{(b \cdots b)}_{x \text{ times}} = a^x \cdot b^x$.

(5) $a^{x\cdot y} = (a^x)^y$

<u>Why?</u> $a^{x\cdot y} = \underbrace{a\cdot a \cdot a \cdots a}_{x\cdot y \text{ times}} =$

$= \underbrace{\underbrace{(a\cdot a \cdots a)}_{x \text{ times}}\cdot\underbrace{(a\cdot a \cdots a)}_{x \text{ times}} \cdots \underbrace{(a\cdot a \cdots a)}_{x \text{ times}}}_{y \text{ times}}$

$= \underbrace{a^x \cdot a^x \cdots a^x}_{y \text{ times}} = (a^x)^y$

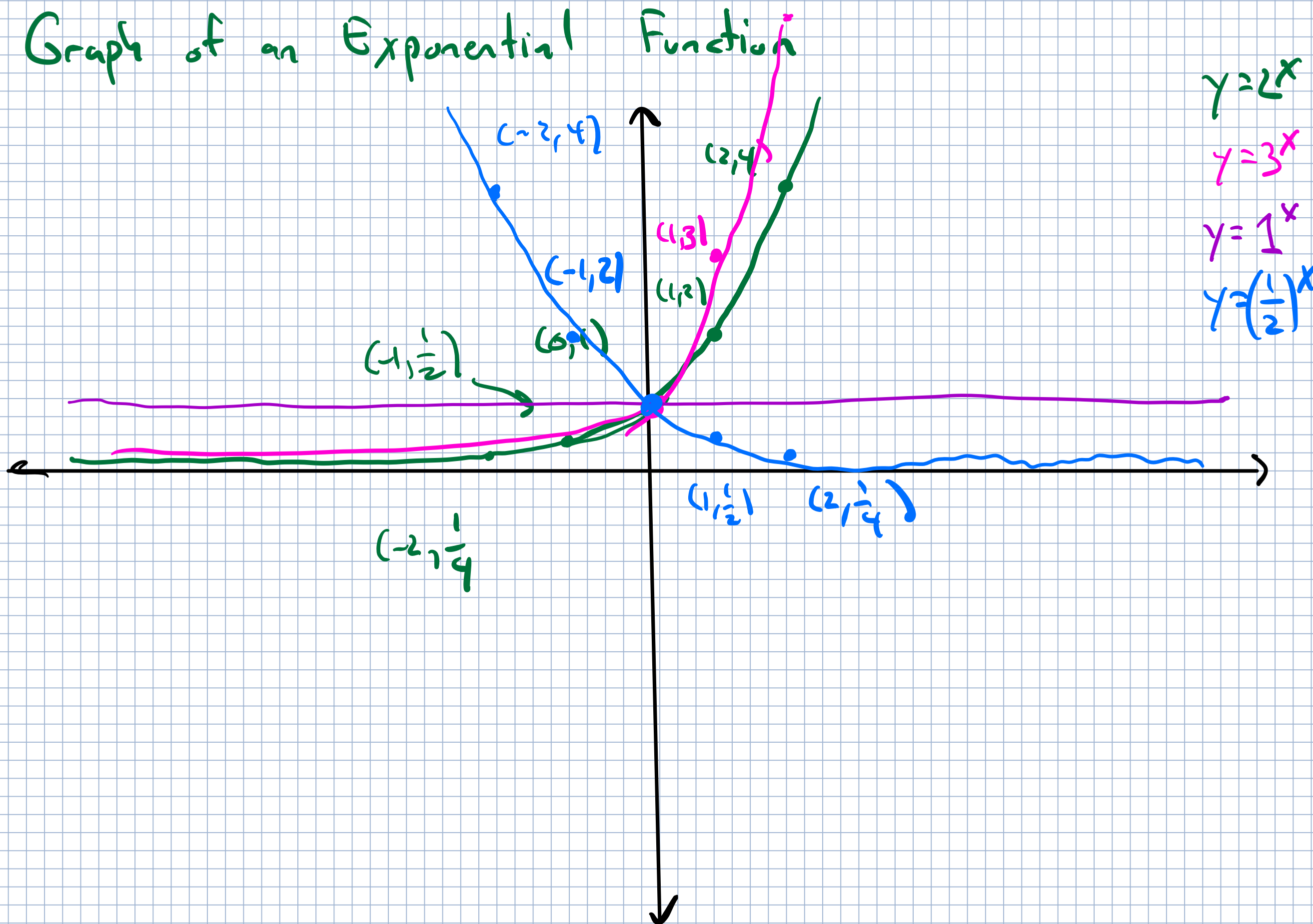
⑥ $a^{1/x} = \sqrt[x]{a}$

Why? $a = a^1 = a^{1/x \cdot x} \overset{⑤}{=} \left(a^{1/x}\right)^x$

Take the xth root of both sides

$\sqrt[x]{a} = a^{1/x}$

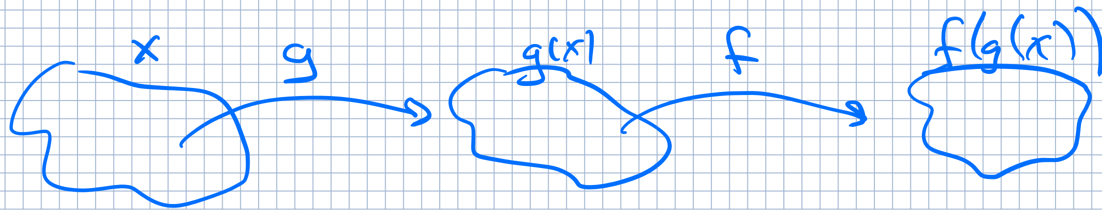
Graph of an Exponential Function



Function Composition

If $f(x)$ and $g(x)$ are functions, their composition is

$$[f \circ g](x) := f(g(x))$$



Ex: $f(x) = 3x+2 \qquad g(x) = x^2$

$$[f\circ g](x) = f(g(x)) = f(x^2) = 3x^2+2$$

$$[g\circ f](x) = g(f(x)) = g(3x+2) = (3x+2)^2$$
$$= 9x^2+12x+4$$

Notice: $f\circ g \neq g\circ f$!

Inverse Functions

Let $f(x)$ be a function. A function $g(x)$ is called the *inverse* of $f(x)$ if: $[f\circ g](x) = [g\circ f](x) = x$.

In other words, the inverse function *undoes* the function $f(x)$.

Ex: $f(x) = x^3$.

The *inverse* of $f(x)$ is $g(x) = \sqrt[3]{x}$.

~~Ex: Find the inverse of~~ $\cancel{f(x) = \frac{3x+2}{x-5}}$.

$\cancel{y = \frac{3x+2}{x-5}}$. ~~Solve for x in terms of y.~~

WARNING! Not all functions have inverses!

Ex: $f(x) = x^3 - x$.

$f(0) = 0^3 - 0 = 0$.

$f(1) = 1^3 - 1 = 0$.

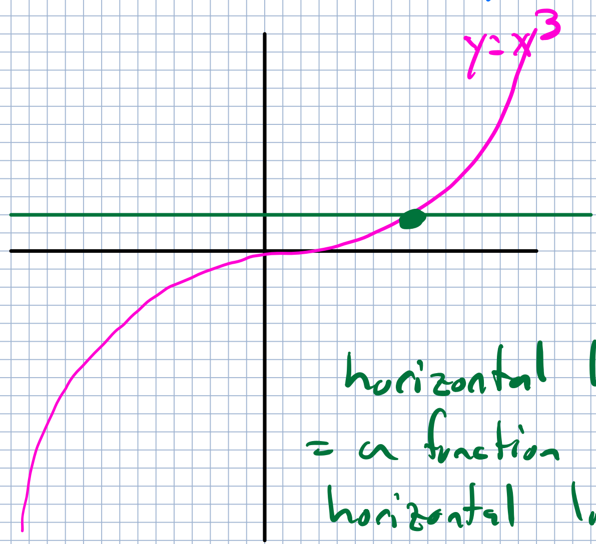
$f(0) = f(1)$.

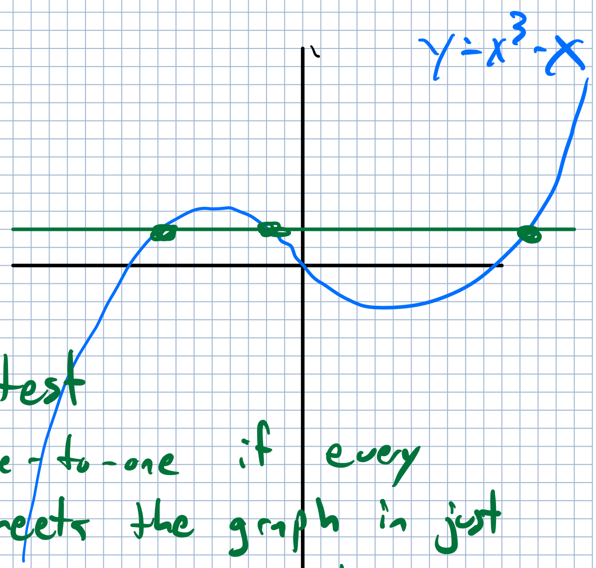
If $f(x)$ had an inverse $g(x)$, then $g(0)$ would have to be <u>the</u> number x that satisfies $f(x)=0$.

There isn't just one number with this property!

The function $f(x)$ is not <u>one-to-one</u>.

A function $f(x)$ is <u>one-to-one</u> if, whenever $f(a)=f(b)$, then $a=b$.





horizontal line test

= a function is one-to-one if every horizontal line meets the graph in just one point.