Exam 2 **IN CLASS** Wednesday, March 9th

→ will cover material between Exam 1 and Monday, Febeuary 28th

Office Hours today end at 1:30

Derivatives of Trig Functions

Ex: Compute the derivative of $\sin(x)$.

Use the formal definition of the derivative.

$$\frac{d}{dx}[\sin(x)] = \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

ANGLE SUM FORMULA

$$= \lim_{h\to 0} \frac{\sin(x)\cos(h) + \cos(x)\sin(h) - \sin(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(x)[\cos(h)-1] + \cos(x)\sin(h)}{h}$$

$$= \sin(x)\underbrace{\boxed{\lim_{h\to 0}\frac{\cos(h)-1}{h}}}_{=0} + \cos(x)\underbrace{\boxed{\lim_{h\to 0}\frac{\sin(h)}{h}}}_{=1}$$

$$= \sin(x)\cdot 0 + \cos(x)\cdot 1 = \cos(x)$$

$$\boxed{\frac{d}{dx}\left[\sin(x)\right] = \cos(x)}$$

Ex: Compute $\frac{d}{dx}\left[\sin(3x^2)\right]$

CHAIN RULE

$$= \frac{d}{dx}[\sin](3x^2) \cdot \frac{d}{dx}\left[3x^2\right]$$

$$= \cos(3x^2) \cdot 6x$$

Ex: Compute $\frac{d}{dx}\left[4x^7 \cdot \sin(x)\right]$

PRODUCT RULE

$$= 4x^7 \cdot \frac{d}{dx}\left[\sin(x)\right] + \sin(x) \cdot \frac{d}{dx}\left[4x^7\right]$$

$$= 4x^7 \cdot \cos(x) + \sin(x) \cdot 4 \cdot 7 \cdot x^6$$

$$= 4x^7 \cdot \cos(x) + 28x^6 \cdot \sin(x)$$

Ex: Compute $\frac{d}{dx}\left[\cos(x)\right]$.

$$\frac{d}{dx}\left[\sin^2(x) + \cos^2(x)\right] = \frac{d}{dx}\left[1\right]$$

CHAIN RULE

$$= 2 \cdot \sin(x) \cdot \frac{d}{dx}\left[\sin(x)\right] + 2 \cdot \cos(x) \cdot \frac{d}{dx}\left[\cos(x)\right] = 0$$

$$= 2\cdot\sin(x)\cdot\cos(x) + 2\cos(x)\cdot\frac{d}{dx}[\cos(x)] = 0$$

$$2\cos(x)\frac{d}{dx}[\cos(x)] = -2\sin(x)\cos(x)$$

$$\frac{d}{dx}[\cos(x)] = \frac{-\cancel{2}\sin(x)\cancel{\cos(x)}}{\cancel{2}\cancel{\cos(x)}} = -\sin(x)$$

$$\boxed{\frac{d}{dx}[\cos(x)] = -\sin(x)}$$

Ex: Compute $\frac{d}{dx}[\tan(x)]$

$$= \frac{d}{dx}\left[\frac{\sin(x)}{\cos(x)}\right] \overset{\text{QUOTIENT RULE}}{=} \frac{\cos(x)\cdot\frac{d}{dx}[\sin(x)] - \sin(x)\cdot\frac{d}{dx}[\cos(x)]}{\cos^2(x)}$$

$$= \frac{\cos(x)\cdot\cos(x) - \sin(x)(-\sin(x))}{\cos^2(x)}$$

$$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$$

$$= \sec^2(x)$$

$$\boxed{\frac{d}{dx}[\tan(x)] = \sec^2(x)}$$

Ex: Compute $\frac{d}{dx}[\sec(x)]$

$$= \frac{d}{dx}\left[\frac{1}{\cos(x)}\right] \overset{\text{QUOTIENT RULE}}{=} \frac{\cos(x)\cdot\frac{d}{dx}[1] - 1\cdot\frac{d}{dx}[\cos(x)]}{\cos^2(x)}$$

$$= \frac{\cancel{\cos(x)\cdot 0} - (-\sin(x))}{\cos^2(x)} = \frac{\sin(x)}{\cos^2(x)}$$

$$= \frac{1}{\cos(x)}\cdot\frac{\sin(x)}{\cos(x)} = \sec(x)\cdot\tan(x)$$

$$\boxed{\frac{d}{dx}[\sec(x)] = \sec(x)\cdot\tan(x)}$$

Ex: Compute $\frac{d}{dx}[\cot(x)]$

$$= \frac{d}{dx}\left[\frac{\cos(x)}{\sin(x)}\right] \overset{\text{QUOTIENT RULE}}{=} \frac{\sin(x)\cdot\frac{d}{dx}[\cos(x)] - \cos(x)\cdot\frac{d}{dx}[\sin(x)]}{\sin^2(x)}$$

$$= \frac{\sin(x)\cdot(-\sin(x)) - \cos(x)\cdot\cos(x)}{\sin^2(x)}$$

$$= \frac{-(\sin^2(x) + \cos^2(x))}{\sin^2(x)} = \frac{-1}{\sin^2(x)}$$

$$= -\csc^2(x)$$

$$\boxed{\frac{d}{dx}[\cot(x)] = -\csc^2(x)}$$

Ex: Compute $\frac{d}{dx}[\csc(x)] = \frac{d}{dx}\left[\frac{1}{\sin(x)}\right]$

$$= \frac{d}{dx}\left[(\sin(x))^{-1}\right] \overset{\text{CHAIN RULE}}{=} -1\cdot(\sin(x))^{-2}\cdot\frac{d}{dx}[\sin(x)]$$

$$= \frac{-\cos(x)}{\sin^2(x)} = -\frac{1}{\sin(x)}\cdot\frac{\cos(x)}{\sin(x)}$$

$$= -\csc(x)\cdot\cot(x)$$

$$\boxed{\frac{d}{dx}[\csc(x)] = -\csc(x)\cot(x)}$$

$$\frac{d}{dx}[\sin(x)] = \cos(x)$$

$$\frac{d}{dx}[\cos(x)] = -\sin(x)$$

$$\frac{d}{dx}[\tan(x)] = \sec^2(x)$$

$$\frac{d}{dx}[\cot(x)] = -\csc^2(x)$$

$$\frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)$$

$$\frac{d}{dx}[\csc(x)] = -\csc(x)\cot(x)$$