Exam 2 Review

Exam 2 IN CLASS this Wednesday, March 9

Same format as last time:

6 multiple choice questions

2 short answer questions ← Show your work!

What's on it?

Limits

- Definition
- Limit Laws
- Limits at Infinity
- Continuous Functions & Intermediate Value Thm
- Sandwich Theorem

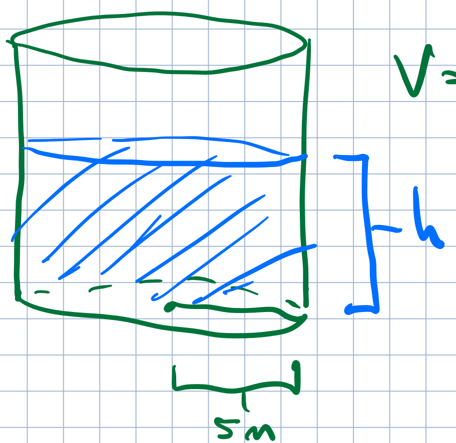
Derivatives

- Definition
- Product Rule
- Sum Rule
- Power Rule
- Quotient Rule
- Chain Rule
- Implicit Differatiation
- Related Rates

Related Rates:

A cylinder-shaped tank has a base radius of 5 m. If water flows out of the tank at a rate of 3 L/hour, how quickly is the depth of the water changing?

① Draw a picture and label everything



V = volume of the water

② Write down what you're given and what you're looking for in terms of the variables you just labeled.

Given: $\frac{dV}{dt} = -3$ L/hour

Want to find: $\frac{dh}{dt}$.

③ Write down an equation that relates the variables

$$V = \pi r^2 h$$

$$V = \pi \cdot 5^2 \cdot h = 25\pi h$$

④ Take derivative of both sides

$$\frac{dV}{dt} = \frac{d}{dt}\left[25\pi h\right] = 25\pi \cdot \frac{dh}{dt}$$

⑤ Plug in

$$-3 = 25\pi \frac{dh}{dt} \qquad \frac{dh}{dt} = \frac{-3}{25\pi} \ \cancel{\text{m/hour}}$$

$$-3\ L/\text{hour} = \underbrace{25\pi}_{m^2}\ \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{-3}{25\pi}\ L/m^2\cdot\text{hours}$$

Implicit Differentiation Ex:

Consider the equation

$$xy = 2y^2 - 6x^3.$$

Find the slope of the tangent line to the graph through the point $(1, 2)$.

In other words, find $\frac{dy}{dx}$.

$$\frac{d}{dx}[xy] = \frac{d}{dx}[2y^2 - 6x^3]$$

PRODUCT RULE   POWER RULE & CHAIN RULE   POWER RULE

$$x\cdot\frac{dy}{dx} + y\cdot\cancel{\frac{dx}{dx}}^{1} = 4y\cdot\frac{dy}{dx} - 6\cdot 3x^2$$

$$x\cdot\frac{dy}{dx} - 4y\frac{dy}{dx} = -y - 18x^2$$

$$(x-4y)\frac{dy}{dx} = -y - 18x^2$$

$$\frac{dy}{dx} = \frac{-y-18x^2}{x-4y}$$

PLUG IN $x=1,\ y=2$

$$\frac{dy}{dx} = \frac{-2-18\cdot 1^2}{1-4\cdot 2} = \frac{-20}{-7} = 20/7$$

Ex! Compute $\frac{d}{dx}\left[\sqrt{\sqrt{x+3}-5}\right]$.

Inside function $g(x)=\sqrt{x+3}$

Outside function $f(x)=\sqrt{x-5}$

$$f(x)=\sqrt{x-5}$$

CHAIN RULE

$$f'(x)=\frac{1}{2}(x-5)^{-1/2}\cdot\frac{d}{dx}[x-5]$$

$$=\frac{1}{2}(x-5)^{-1/2}$$

$$=\frac{1}{2\sqrt{x-5}}$$

$$g(x)=\sqrt{x+3}$$

CHAIN RULE

$$g'(x)=\frac{1}{2}(x+3)^{-1/2}\cdot\frac{d}{dx}[x+3]$$

$$=\frac{1}{2}(x+3)^{-1/2}$$

$$=\frac{1}{2\sqrt{x+3}}$$

$$\frac{d}{dx}[f(g(x))]\overset{\text{CHAIN RULE}}{=}f'(g(x))\cdot g'(x)$$

$$=\frac{1}{2\sqrt{\sqrt{x+3}-5}}\cdot\frac{1}{2\sqrt{x+3}}$$

Sandwich Theorem Ex:

Compute $\lim_{x\to 0} x\cdot\sin\left(\frac{1}{x}\right)$.

$$-1\le\sin\left(\frac{1}{x}\right)\le 1$$

CAUTION!

$$\cancel{-x\le x\sin\left(\frac{1}{x}\right)\le x}$$

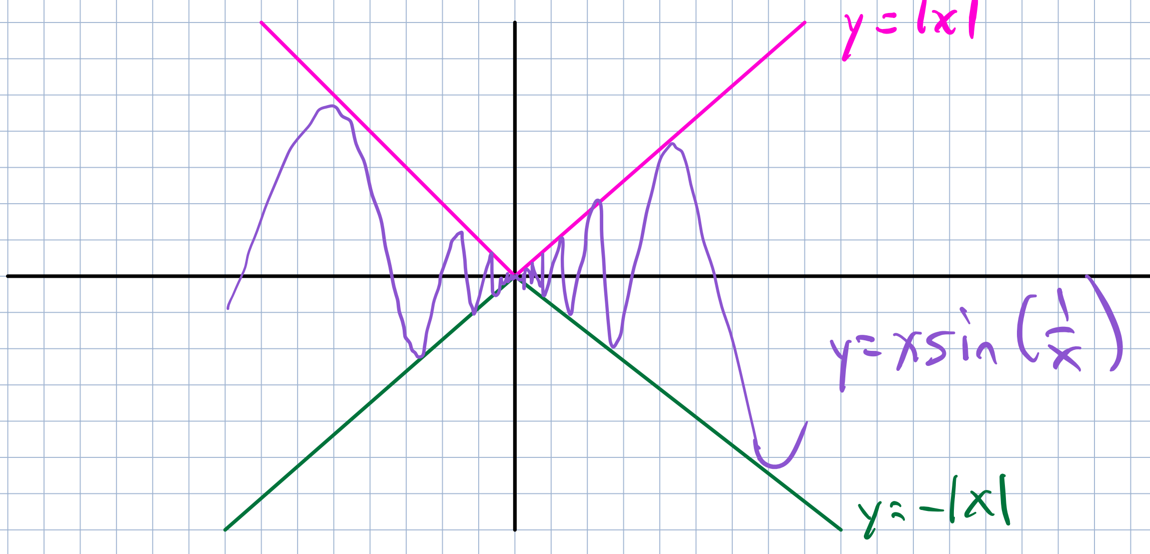
NOT TRUE b/c x might be negative

$$-|x| \le x\sin\left(\frac{1}{x}\right) \le |x|.$$

$$\lim_{x\to 0} |x| = |0| = 0$$ b/c $|x|$ is a continuous function

$$\lim_{x\to 0} -|x| = -|0| = 0$$ b/c $-|x|$ is a continuous function

By the Sandwich Theorem, since $\lim_{x\to 0} |x| = \lim_{x\to 0} -|x| = 0$, we have $\lim_{x\to 0} x \cdot \sin\left(\frac{1}{x}\right) = 0$.



Limits at Infinity Ex:

$$\lim_{x\to\infty} \frac{3x^3+5x^2-7}{6x^3+4x+3} \cdot \frac{1/x^3}{1/x^3}$$

Know!

$$\lim_{x\to\pm\infty} \frac{1}{x} = 0.$$

$$\lim_{x\to\pm\infty} c = c$$

$$= \lim_{x\to\infty} \frac{3 + \frac{5}{x} - \frac{7}{x^3}}{6 + \frac{4}{x^2} + \frac{3}{x^3}}$$

$$\lim_{x\to -\infty} e^x = 0.$$

$$= \frac{\lim_{x\to\infty} 3 + 5\lim_{x\to\infty}\frac{1}{x} - 7\left(\lim_{x\to\infty}\frac{1}{x}\right)^3}{\lim_{x\to\infty} 6 + 4\left(\lim_{x\to\infty}\frac{1}{x}\right)^2 + 3\left(\lim_{x\to\infty}\frac{1}{x}\right)^3}$$

$$= \frac{3}{6} = \frac{1}{2}$$