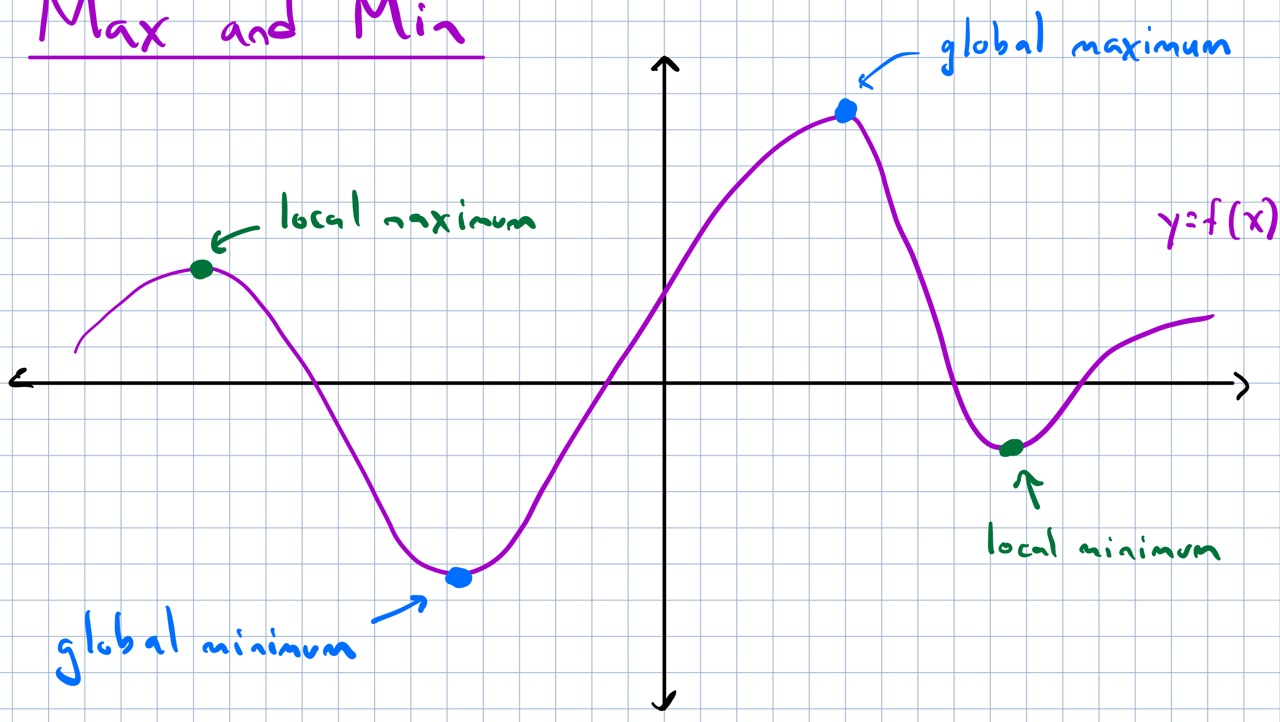
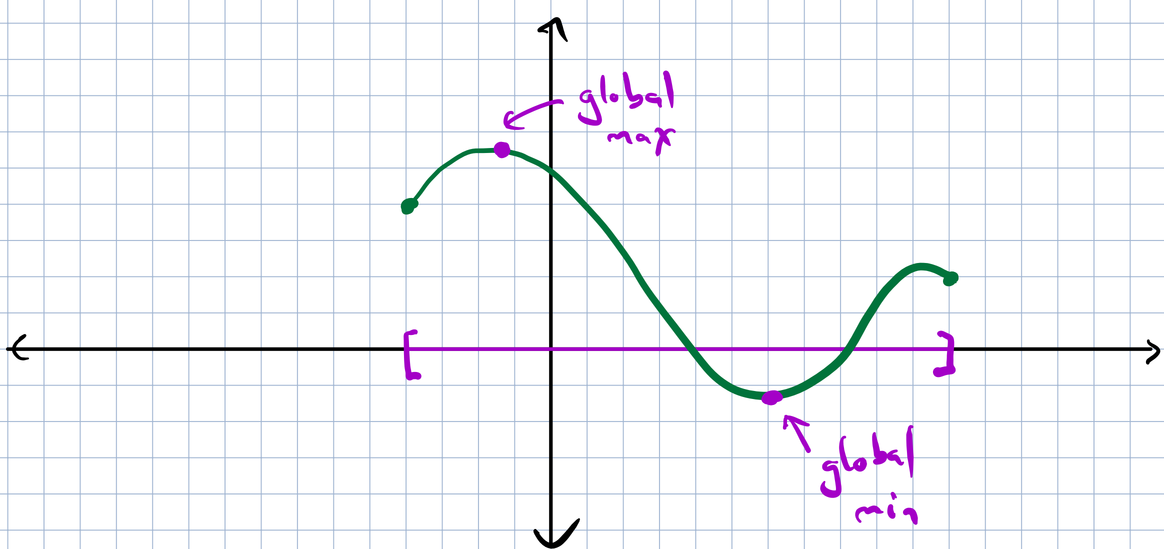
Max and Min

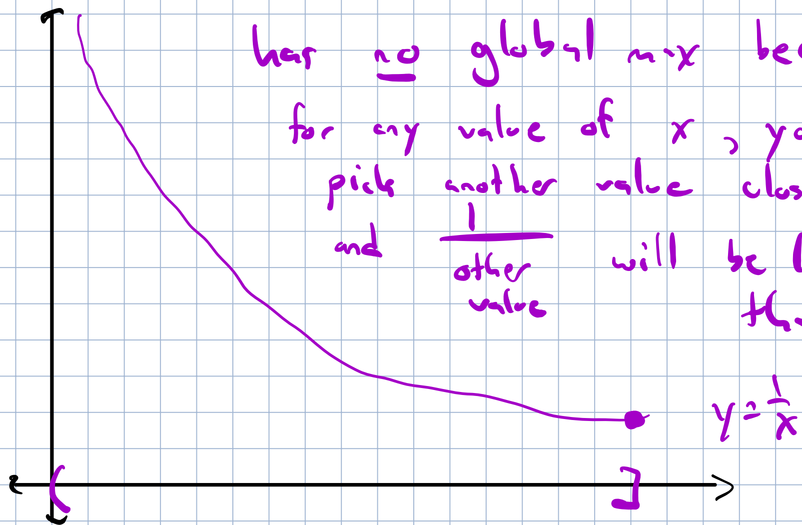


Important Fact: Any continuous function on a closed interval has a global maximum and a global minimum.



This is not true if the interval is not closed!

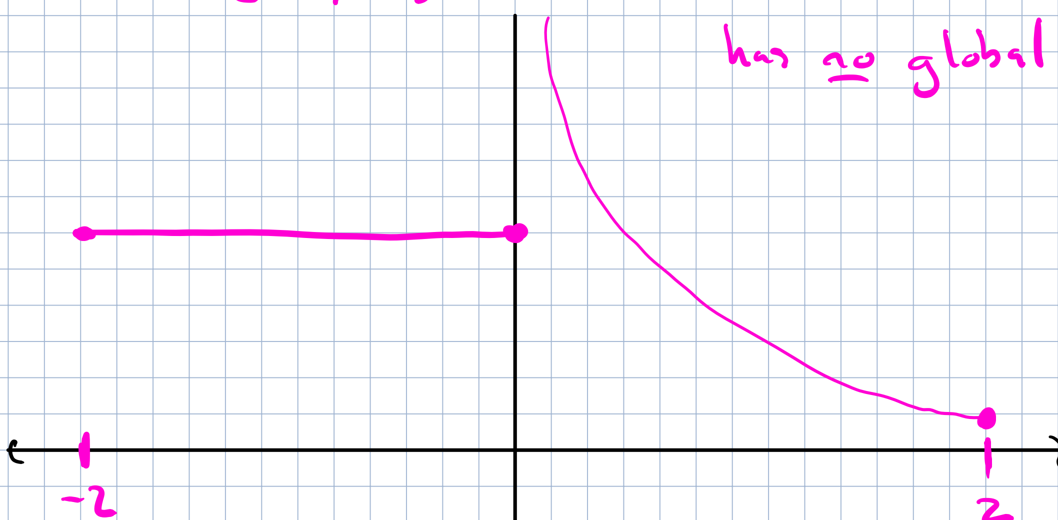
Ex: Consider $y = \frac{1}{x}$ on $(0, 2]$.



has <u>no</u> global max because for any value of x, you can pick another value closer to 0 and $\frac{1}{\text{other value}}$ will be larger than $\frac{1}{x}$.

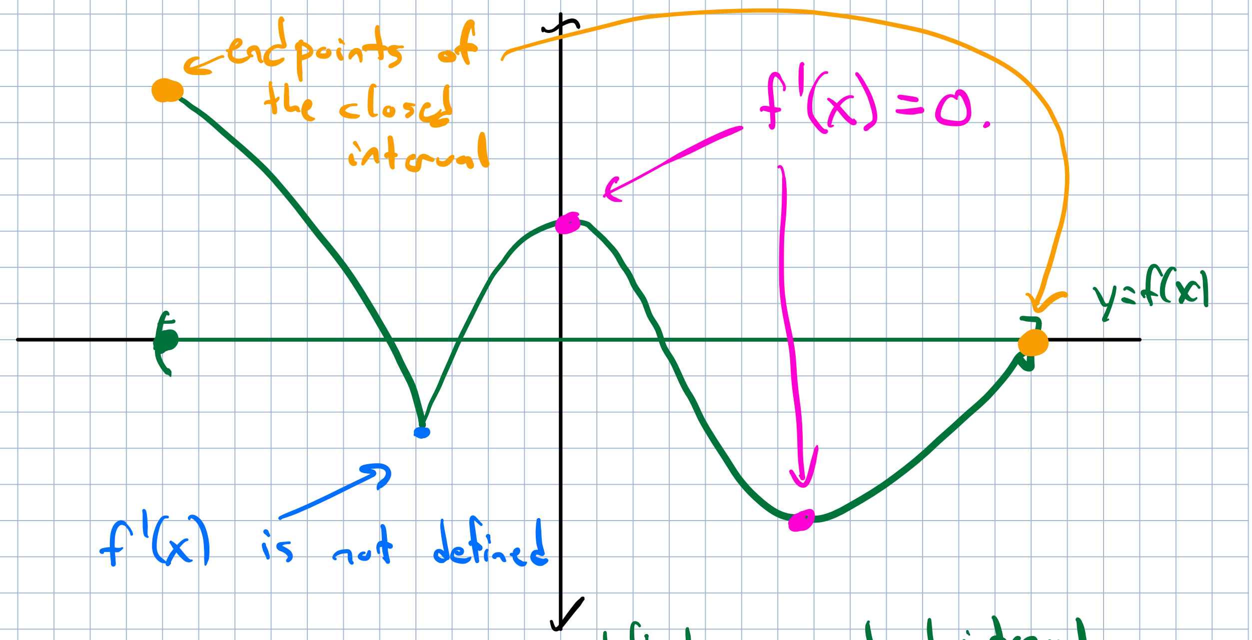
This is also not true if the function is not continuous.

<u>Ex</u>: Consider $f(x) = \begin{cases} \frac{1}{x} & \text{when } x > 0 \\ 1 & \text{when } x \leq 0 \end{cases}$ on $[-2, 2]$.



has <u>no</u> global max.

Where could the $\underset{\text{local}}{\wedge}$ max or $\underset{\text{local}}{\wedge}$ min occur?



A continuous function $f(x)$ defined on a closed interval can have a local max or min at c if:

1. $f'(c) = 0$
2. $f'(c)$ is not defined
3. c is one of the 2 endpoints of the interval.

Ex: Find all of the local maxima and minima of $f(x) = |x^2-4|$ on $[-2.5, 2.5]$. Which of these are global maxima and minima?

$$f(x) = \begin{cases} x^2-4 & \text{when } x^2-4 \geq 0 \\ 4-x^2 & \text{when } x^2-4 < 0 \end{cases}$$

$x^2-4 < 0$ when $x^2 < 4$ when $-2 < x < 2$

$$f(x) = \begin{cases} x^2-4 & \text{if} \quad x \le -2 \text{ or } x \ge 2 \\ 4-x^2 & \text{if} \quad -2 < x < 2 \end{cases}$$

$$f'(x) = \begin{cases} 2x & \text{if} \quad x < -2 \text{ or } x > 2 \\ -2x & \text{if} \quad -2 < x < 2. \end{cases}$$

(1) when is $f'(x) = 0$?

when $-2x = 0$, which is $x = 0$.

(2) when is $f'(x)$ undefined?

at $x = -2$, $\left.\begin{array}{l} 2x = -4 \\ -2x = 4 \end{array}\right]$ don't match up!

at $x = 2$ $\left.\begin{array}{l} 2x = 4 \\ -2x = -4 \end{array}\right]$ don't match up!

(3) what are the endpoints?

$x = -2.5$ and $x = 2.5$

Local extrema could occur at $x = -2.5, -2, 0, 2, 2.5$.

Which are the global maxima + minima?

$f(-2.5) = |(-2.5)^2 - 4| = |6.25 - 4| = 2.25$

$f(-2) = |(-2)^2 - 4| = |4 - 4| = 0 \leftarrow$ global min

$f(0) = |0^2 - 4| = |-4| = 4 \leftarrow$ global max

$f(2) = 0 \leftarrow$ global min

$f(2.5) = 2.25$

global max at $x=0$ (where $f'=0$.)