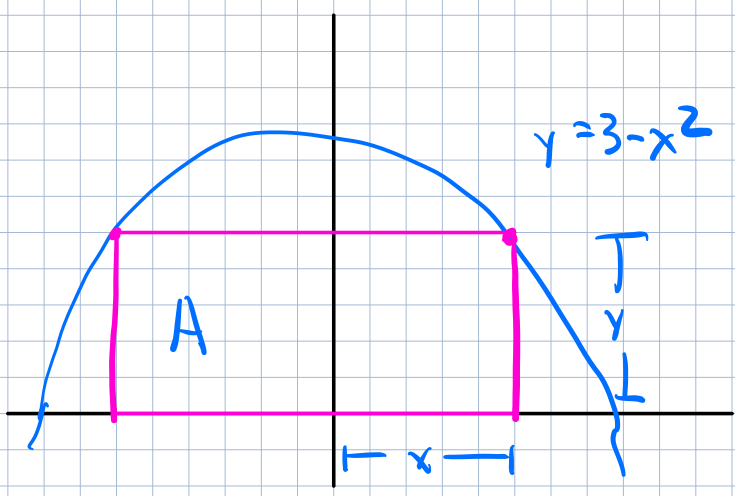
Exam 3 IN CLASS Monday, April 11

Final Project due on Canvas Monday, April 18

Optimization

Ex: A rectangle has its base on the x-axis and its upper 2 corners on the parabola $y = 3 - x^2$. What is the largest area the rectangle can have?

(1) Draw a picture + label everything



(2) What are we trying to maximize? What are we given?

Maximize A.

Given: $y = 3 - x^2$

(3) Write the function that we want to optimize as a function in 1 variable.

$$A = 2xy = 2x(3 - x^2)$$
$$= 6x - 2x^3.$$

$y = 0$

$3 - x^2 = 0$ $\quad x^2 = 3$

(4) Where are the endpoints? x is in the interval $[0, \sqrt{3}]$.

(5) Where is $\frac{dA}{dx}=0$? Where is $\frac{dA}{dx}$ undefined?

$$\frac{dA}{dx} = \frac{d}{dx}\left[6x-2x^3\right] = 6 - 6x^2 = 0$$

$$6 = 6x^2$$

$$1 = x^2$$

$$\pm 1 = x$$

$x=-1$ is not in the interval $[0,\sqrt{3}]$, so we're left with $x=1$.

(6) At which of these could the max occur?

$$A(0) = 6\cdot 0 - 2\cdot 0^3 = 0$$

$$A(\sqrt{3}) = 6\cdot\sqrt{3} - 2\cdot(\sqrt{3})^3 = 6\cdot\sqrt{3} - 6\cdot\sqrt{3} = 0$$

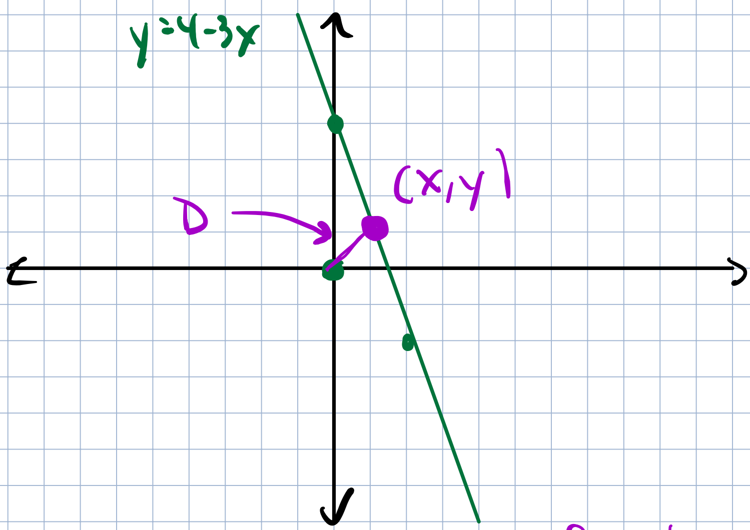
$$A(1) = 6\cdot 1 - 2\cdot 1^3 = 6-2 = 4$$

The biggest of these 3 values occurs at $x=1$, so the maximal possible area is when $x=1$. $y = 3-x^2 = 3-1^2 = 2$.

$$A = 2xy = 2\cdot 1\cdot 2 = 4.$$

Ex: Find the point on the line $y=4-3x$ that is closest to the origin.

① Draw a picture

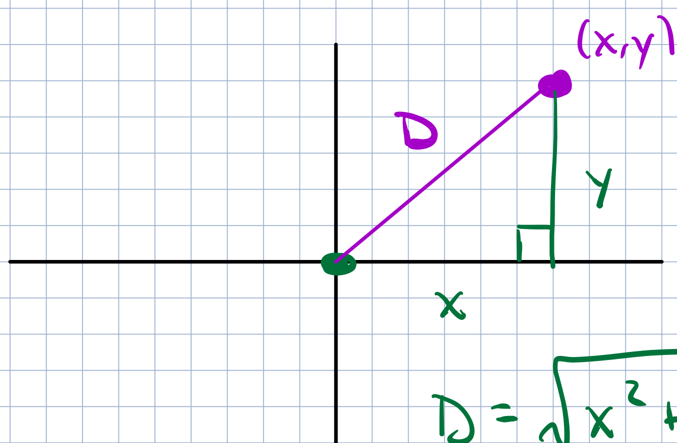


② What function do we want to minimize? What are we given?

Minimize D

Given: $y=4-3x$.

③ Write D as a function of a single variable.



$$D^2 = x^2+y^2$$

$$D=\sqrt{x^2+y^2}$$

$$D=\sqrt{x^2+(4-3x)^2}$$

Instead of finding where D obtains its min, I'm going to find where D^2 obtains its min.

$$F = D^2 = x^2+(4-3x)^2 = x^2+16-24x+9x^2$$

$$=10x^2-24x+16$$

(4) Where are the endpoints?

In this problem, there are no endpoints!

We're not guaranteed that F will have a global min.

(5) Where is $\frac{dF}{dx} = 0$? Where is $\frac{dF}{dx}$ undefined?

$$\frac{dF}{dx} = \frac{d}{dx}\left[16x^2 - 24x + 16\right]$$

$$= 20x - 24 = 4(5x-6) = 0$$

when $5x - 6 = 0$

$5x = 6 \qquad x = \frac{6}{5}$

$$y = 4 - 3x = 4 - \frac{18}{5} = \frac{20}{5} - \frac{18}{5} = \frac{2}{5}$$

$(x, y) = \left(\frac{6}{5}, \frac{2}{5}\right)$.

(6) How do we know that F has a global min at $x = 6/5$?

First Derivative Test.

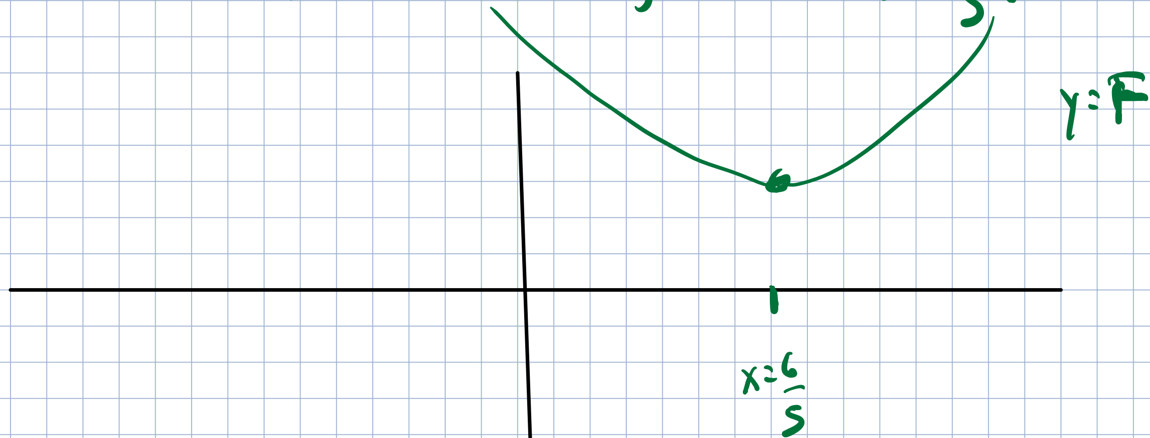
$\frac{dF}{dx} = 4(5x-6)$.

If $x > \frac{6}{5}$, then $\frac{dF}{dx} > 0$

If $x < \frac{6}{5}$, then $\frac{dF}{dx} < 0$.

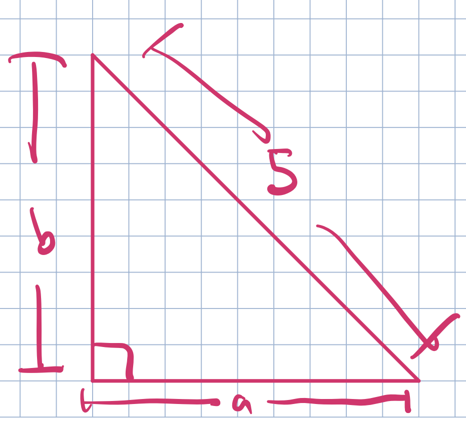
This means F is increasing when $x > \frac{6}{5}$

F is decreasing when $x < \frac{6}{5}$.



So $x = \frac{6}{5}$ is a global min for F.

Ex: Suppose that a and b are the side lengths of a right triangle with hypotenuse 5 cm. What is the largest perimeter possible?



Maximize perimeter P.

$$P = a + b + 5$$

$$a^2 + b^2 = 5^2 = 25$$

$b^2 = 25 - a^2 \qquad b = \sqrt{25 - a^2}$

$P = a + \sqrt{25 - a^2} + 5$

<u>Endpoints</u>: a is in the interval $[0, 5]$.

$$\frac{dP}{da} = \frac{d}{da}\left[a + \sqrt{25 - a^2} + 5\right]$$

$$= 1 + \frac{-\cancel{2}a}{\cancel{2}\sqrt{25 - a^2}} = 1 - \frac{a}{\sqrt{25 - a^2}} = 0$$

$$1 = \frac{a}{\sqrt{25 - a^2}} \qquad \sqrt{25 - a^2} = a$$

$$25 - a^2 = a^2$$

$$25 = 2a^2$$

$$\frac{25}{2} = a^2$$

$$\frac{5}{\sqrt{2}} = a = b.$$

$P(0) = 0 + \sqrt{25 - 0^2} + 5 = 10$

$P(5) = 5 + \sqrt{25 - 5^2} + 5 = 10$

$$P\left(\frac{5}{\sqrt{2}}\right) = \frac{5}{\sqrt{2}} + \sqrt{25 - \left(\frac{5}{\sqrt{2}}\right)^2} + 5$$

$$= \frac{5}{\sqrt{2}} + \frac{5}{\sqrt{2}} + 5 = \frac{10}{\sqrt{2}} + 5 > 10.$$

P obtains its max when $a = b = \frac{5}{\sqrt{2}}$.