MA 137 Lecture 3 — Logarithms

First Exam — IN CLASS February 2

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Exponential Functions are good models for population growth

Ex: $F(x)$ = number of rabbits in a population after x months.

$$F(x) = 2^x.$$

Q: How many rabbits will there be after 5 months?

$$F(5) = 2^5 = 32.$$

Q: How long will it ~~be~~ take for the rabbit population to reach 100?

For what value of x is $F(x) = 100$?

The second question is asking for an inverse function to $F(x)$.

Ex: Find the inverse of $F(x) = \dfrac{3x+2}{x-7}$.

$$y = \frac{3x+2}{x-7}.$$

Solve for x in terms of y.

$$yx - 7y = 3x + 2$$

$$yx = 3x + 7y + 2$$

$$yx - 3x = 7y + 2$$

$$(y-3)x = 7y + 2$$

$$x = \frac{7y+2}{y-3}$$

The inverse function to $F(x)$ is the function $\dfrac{7x+2}{x-3}$.

The inverse of an exponential function is called a <u>logarithm</u>.

For a positive number a, $a \neq 1$, the inverse of the exponential function a^x is called the <u>logarithm base a</u>, and denoted $\log_a(x)$.

In other words, if $y = a^x$, then $\log_a(y) = x$.

<u>Ex</u>: $\log_2(16) = 4$ because $2^4 = 16$.

<u>Ex</u>: $\log_{10}(1000) = 3$ because $10^3 = 1000$.

In general, $\log_{10}(N) \sim$ the number of digits in N.

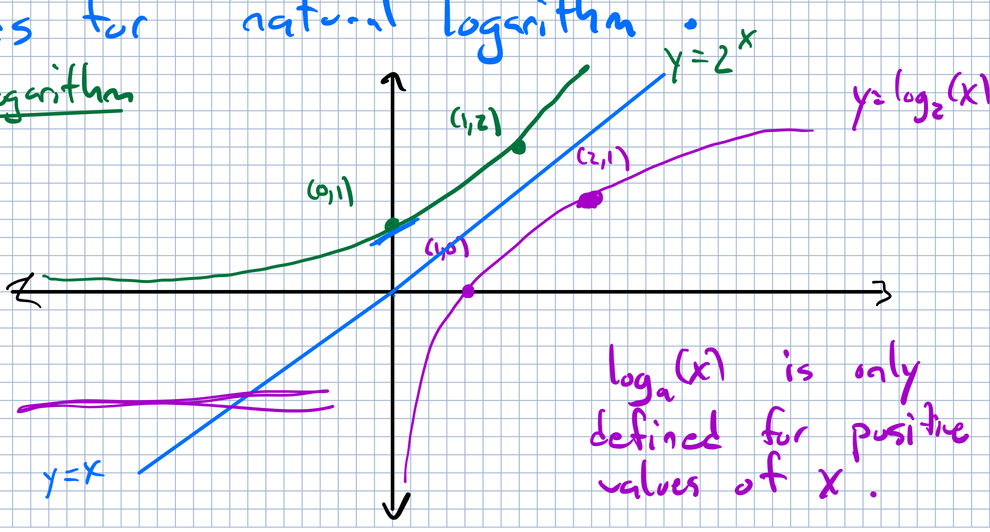
In this class, if I write $\log(x)$, I mean $\log_{10}(x)$.

If I write $\ln(x)$, I mean $\log_e(x)$, where $e \approx 2.718281828\ldots$

The reason that mathematicians prefer base e requires some calculus to understand.

$\ln(x)$ stands for "natural logarithm".

<u>Graph of a Logarithm</u>



$\log_a(x)$ is only defined for positive values of x.

Properties of Logs

① $\log_a(1) = 0$

Why? $a^0 = 1$. $0 = \log_a(a^0) = \log_a(1)$

② $\log_a(x) + \log_a(y) = \log_a(x \cdot y)$.

Why? $a^{\log_a(x)+\log_a(y)} = \underbrace{a^{\log_a(x)}} \cdot \underbrace{a^{\log_a(y)}} = x \cdot y$

$= a^{\log_a(x \cdot y)}$

So $\log_a(x) + \log_a(y) = \log_a(x \cdot y)$.

③ $\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$.

Why? $a^{\log_a(x)-\log_a(y)} = \frac{a^{\log_a(x)}}{a^{\log_a(y)}} = \frac{x}{y}$

$= a^{\log_a\left(\frac{x}{y}\right)}$

So $\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$.

④ $\log_a(x^n) = n \cdot \log_a(x)$

Why? $a^{n \cdot \log_a(x)} = \left(a^{\log_a(x)}\right)^n = x^n = a^{\log_a(x^n)}$

So $n \cdot \log_a(x) = \log_a(x^n)$.

Change of Base Formula:

⑤ $\log_a(x) = \frac{\ln(x)}{\ln(a)}$.

Why? $a^x = \left(e^{\ln(a)}\right)^x = e^{\ln(a) \cdot x}$