Exam 3 IN CLASS Monday, April 11

What's on the Exam?

Derivatives

- derivatives of trig functions
- derivatives of exponentials
- derivatives of logs / logarithmic differentiation

Applications of Derivatives

- Graphing
  - maxima + minima
  - increasing + decreasing
  - Mean Value Theorem
  - inflection points + concavity
- Optimization
- L'Hôpital's Rule

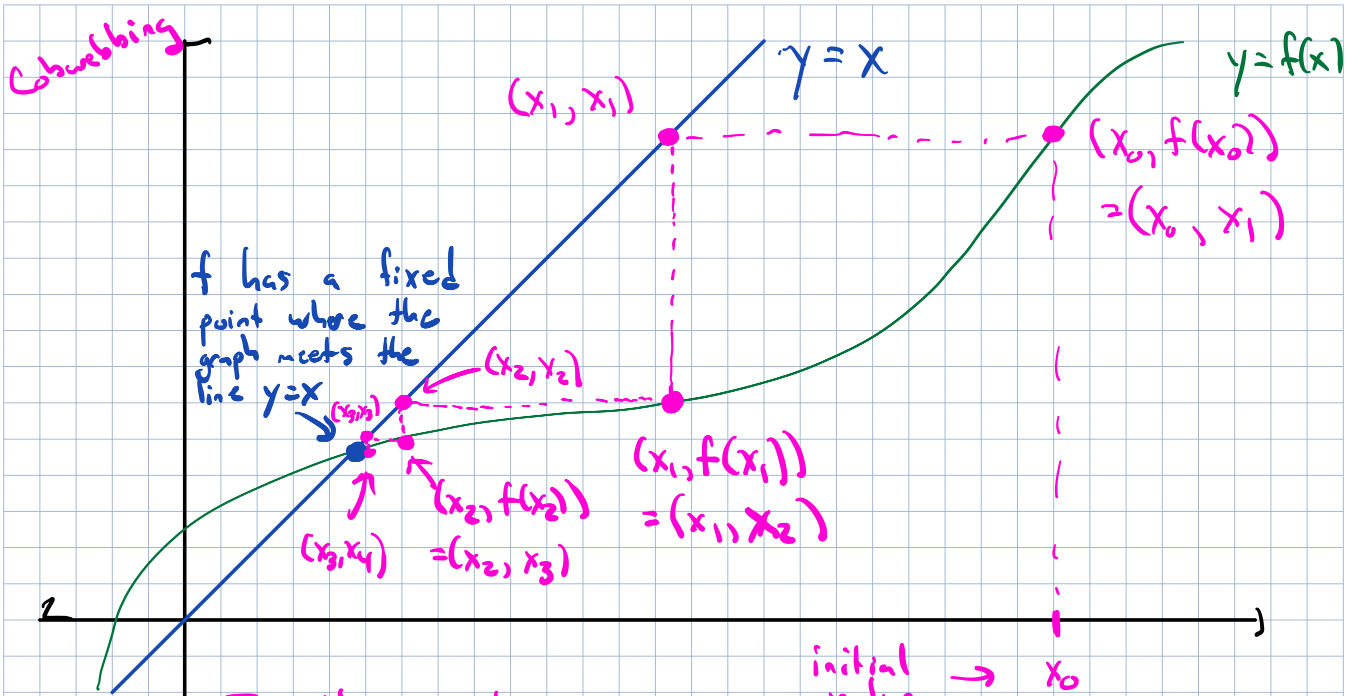
Today's topic will NOT be on the exam

Recursively defined sequences

$a_0, a_1, a_2, a_3, \ldots$

$$a_{n+1} = f(a_n)$$

If the recursively defined sequence has a limit, then that limit must be a fixed point for the recursion. In other words, it must satisfy $x = f(x)$.



In this picture, we see that the x_n's are getting closer and closer to the fixed point. So that fixed point is <u>locally stable</u>.

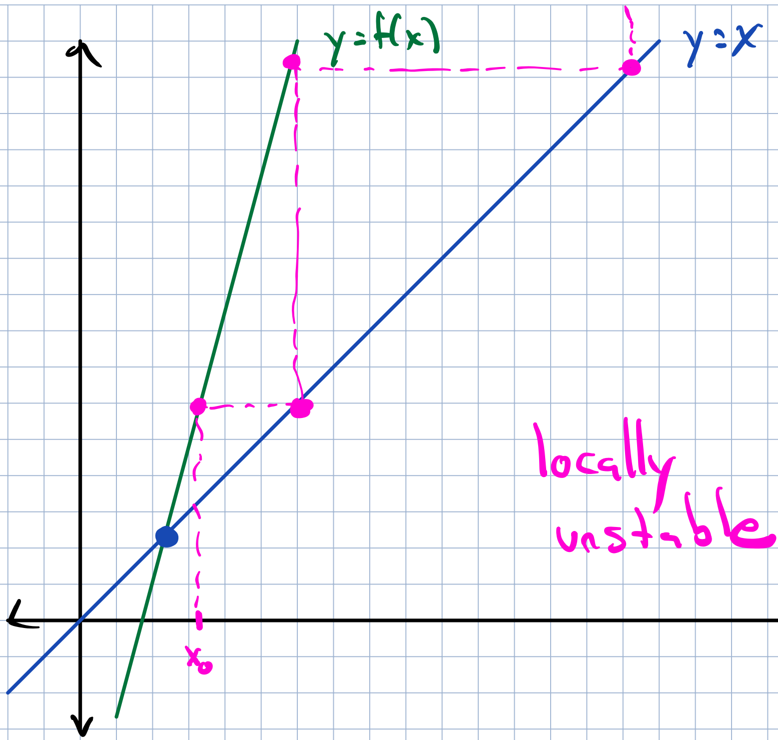
If you look locally enough, a graph of $f(x)$ near a looks like a line with slope $f'(a)$.

It's enough to do this cobwebbing procedure in the case where $f(x)$ is linear.

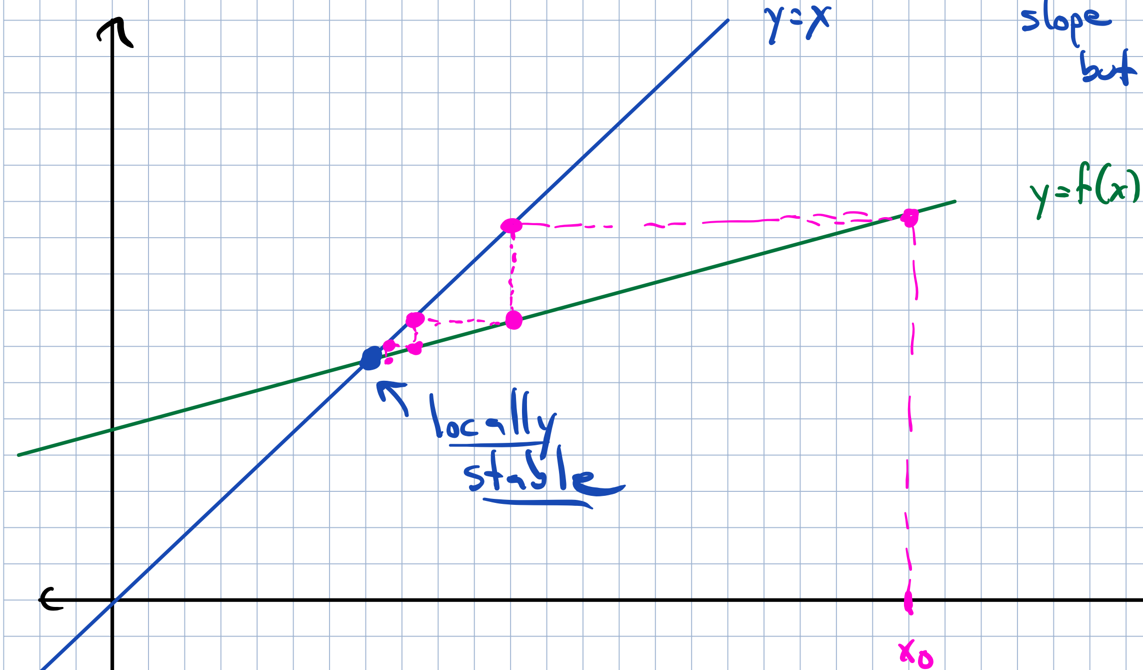
4 cases: slope of the tangent line could be

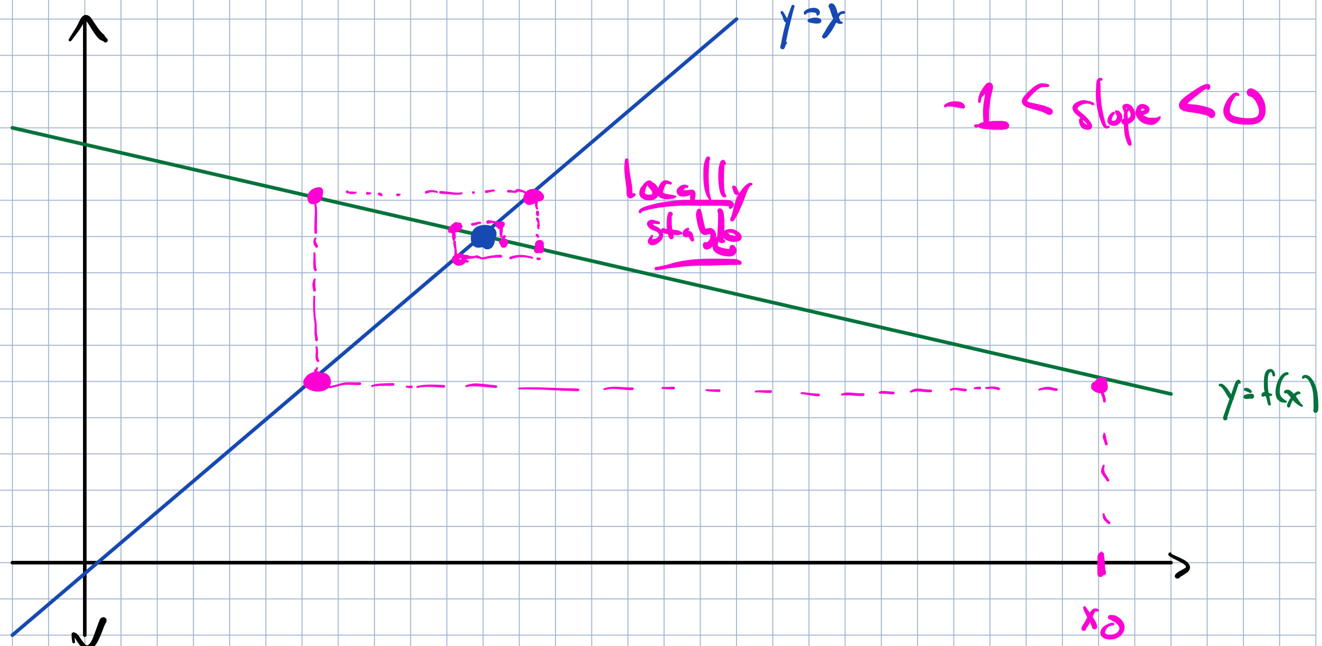
$f'(a) > 1$, $0 \leq f'(a) < 1$, $-1 \leq f'(a) < 0$, $f'(a) < -1$

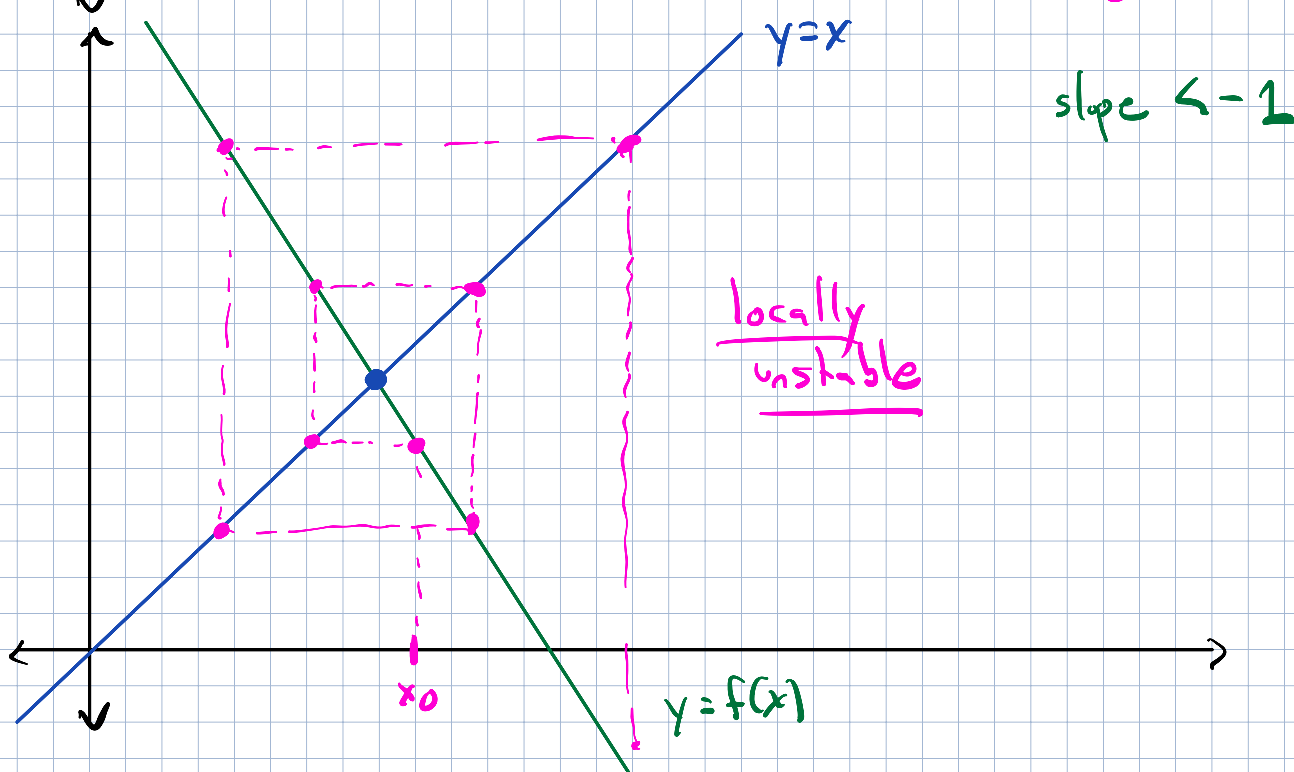
slope > 1



slope is positive but < 1







Let a be a fixed point for the recursion $a_{n+1} = f(a_n)$.

If $|f'(a)| < 1$, then a is locally stable.

If $|f'(a)| > 1$, then a is locally unstable.

Ex: $a_{n+1} = \frac{1}{4} a_n^2 - \frac{5}{4}$. Find the fixed points and characterize their local stability.

$$x = \frac{1}{4}x^2 - \frac{5}{4}.$$

$$0 = \frac{1}{4}x^2 - x - \frac{5}{4}$$

$$0 = x^2 - 4x - 5$$
$$= (x-5)(x+1)$$

Two fixed points: $x = -1$, $x = 5$.

$$f(x) = \frac{1}{4}x^2 - \frac{5}{4}.$$

$$f'(x) = \frac{1}{2}x$$

$$f'(-1) = \frac{1}{2}\cdot(-1) = -\frac{1}{2}. \qquad |f'(-1)| = \left|-\frac{1}{2}\right| = \frac{1}{2} < 1$$

so $x=-1$ is a locally stable fixed point.

$$f'(5) = \frac{1}{2}\cdot 5 = \frac{5}{2} \qquad |f'(5)| = \left|\frac{5}{2}\right| = \frac{5}{2} > 1$$

so $x=5$ is a locally unstable fixed point.

Ex: $a_{n+1} = \dfrac{a_n}{\frac{1}{10} + a_n}$. Find the fixed points and characterize their local stability.

$$x = \frac{x}{\frac{1}{10} + x}$$

$$x\left(\frac{1}{10} + x\right) = x$$

$$\frac{1}{10}x + x^2 = x$$

$$x^2 - \frac{9}{10}x = 0$$

$$x\left(x-\frac{9}{10}\right)=0$$

Two fixed points: $x=0, \frac{9}{10}$.

$$f(x)=\frac{x}{\frac{1}{10}+x}$$

$$f'(x)=\frac{\left(\frac{1}{10}+x\right)\cdot\frac{d}{dx}[x]-x\cdot\frac{d}{dx}\left[\frac{1}{10}+x\right]}{\left(\frac{1}{10}+x\right)^2}$$

$$=\frac{\left(\frac{1}{10}+x\right)\cdot 1-x\cdot 1}{\left(\frac{1}{10}+x\right)^2}$$

$$=\frac{\frac{1}{10}+x-x}{\left(\frac{1}{10}+x\right)^2}=\frac{\frac{1}{10}}{\left(\frac{1}{10}+x\right)^2}$$

$$f'(0)=\frac{\frac{1}{10}}{\left(\frac{1}{10}+0\right)^2}=\frac{\frac{1}{10}}{\frac{1}{100}}=\frac{100}{10}=10>1$$

so $x=0$ is a locally unstable fixed point.

$$f'\left(\frac{9}{10}\right)=\frac{\frac{1}{10}}{\left(\frac{1}{10}+\frac{9}{10}\right)^2}=\frac{\frac{1}{10}}{1^2}=\frac{1}{10}<1.$$

so $x=\frac{9}{10}$ is a locally stable fixed point.