Final Project due Monday April 18
on Canvas

Antiderivatives

Ex: Let $f(x) = x^2 + 3$.
Find a function whose derivative is $f(x)$.

$g(x) = \frac{1}{3}x^3 + 3x$

Let's check that $g'(x) = f(x)$.

$$g'(x) = \frac{d}{dx}\left[\frac{1}{3}x^3 + 3x\right]$$
$$= \frac{1}{3}\cdot 3x^2 + 3 = x^2 + 3 \quad \checkmark$$

Def: An antiderivative of a function $f(x)$ is a function whose derivative is $f(x)$.

Note: A function can have more than 1 antiderivative!

Ex: $f(x) = x^2 + 3$.
$\frac{1}{3}x^3 + 3x$ is an antiderivative of $f(x)$

$\frac{1}{3}x^3 + 3x + 7$ is also an antiderivative of $f(x)$!

$\frac{1}{3}x^3 + 3x + 5$

are also antiderivatives of $f(x)$.

$\frac{1}{3}x^3 + 3x + \pi$

Ex: Find all antiderivatives of the constant function $f(x) = 0$.

Let $g(x)$ be an antiderivative of $f(x) = 0$.

Let's show that $g(x)$ is a constant function.

Choose numbers a and b. We want to show that $g(a) = g(b)$.

By the MVT, there is a number c between a and b such that

$$0 = f(c) = g'(c) = \frac{g(b) - g(a)}{b - a}$$

$$0 = g(b) - g(a)$$

$$g(a) = g(b).$$

We're done!

Every antiderivative of $f(x) = 0$ is a constant function.

Now, let's see that for ~~my~~ function $f(x)$, any two antiderivatives of $f(x)$ differ by a constant.

Why? Let $g_1(x)$, $g_2(x)$ be antiderivatives of $f(x)$.

This means $g_1'(x) = f(x)$ $\quad g_2'(x) = f(x)$.

$$\frac{d}{dx}\left[g_1(x) - g_2(x)\right] = g_1'(x) - g_2'(x)$$
$$= f(x) - f(x) = 0$$

So $g_1(x) - g_2(x)$ is a constant C.

$$g_1(x) - g_2(x) = C$$

$$g_1(x) = g_2(x) + C.$$

<u>Ex</u>: Find ~~a~~ all functions whose derivative is $f(x) = x^2 + 3$.

We already found one function whose derivative is $f(x)$.

Namely, $g(x) = \frac{1}{3}x^3 + 3x$

<u>All</u> antiderivatives of $f(x)$ are of the form:

$\frac{1}{3}x^3 + 3x + C$

Table of Antiderivatives:

$f(x)$	All antiderivatives of $f(x)$
0	C
m (constant function)	$mx + C$
x^n $(n \neq -1)$	$\frac{1}{n+1}x^{n+1} + C$
$\frac{1}{x}$	$\ln(x) + C$
e^x	$e^x + C$
$\cos(x)$	$\sin(x) + C$
$\sin(x)$	$-\cos(x) + C$
$\sec^2(x)$	$\tan(x) + C$
$\csc^2(x)$	$-\cot(x) + C$
$\sec(x)\tan(x)$	$\sec(x) + C$
$\csc(x)\cot(x)$	$-\csc(x) + C$

Ex: Let $f(x) = 3e^x + \cos(x) - 7x^5 + 3x$.

Find ALL antiderivatives of $f(x)$.

$$3e^x + \sin(x) - 7 \cdot \frac{1}{6}x^6 + 3 \cdot \frac{1}{2}x^2 + c$$

$$3e^x + \sin(x) - \frac{7}{6}x^6 + \frac{3}{2}x^2 + c$$

Let's check our answer.

$$\frac{d}{dx}\left[3e^x + \sin(x) - \frac{7}{6}x^6 + \frac{3}{2}x^2 + c\right]$$

$$= 3e^x + \cos(x) - \frac{7}{6} \cdot 6x^5 + \frac{3}{2} \cdot 2x$$

$$= 3e^x + \cos(x) - 7x^5 + 3x \quad = f(x) \checkmark$$

Ex: Let $f(x) = 7\sec^2(x) + \frac{13}{x} - x^4$.

Find all antiderivatives of $f(x)$.

$$7\tan(x) + 13 \cdot \ln(x) - \frac{1}{5}x^5 + C$$

Let's check that our answer is correct

$$\frac{d}{dx}\left[7\tan(x) + 13\ln(x) - \frac{1}{5}x^5 + C\right]$$

$$= 7\sec^2(x) + 13 \cdot \frac{1}{x} - \cancel{\frac{1}{5}} \cdot \cancel{5}x^4 \quad = f(x) \checkmark$$

If you wanted to find an antiderivative of:-

$x \cdot \sin(x)$ you need more tools.

or $\ln(x)$

or $\sec(x)$