Final exam Wednesday, May 4 10:30 AM – 12:30 PM
here

If you're going to miss the final or have any other outstanding coursework, you need to email me on or before May 4th at dave.h.jensen@gmail.com

I will be dropping the 3 lowest quiz grades

Course evaluations are now open

Definite Integral

Def: Let $f(x)$ be a function, a and b numbers, and define $\Delta x = \frac{b-a}{n}$.

The definite integral of $f(x)$ from a to b is

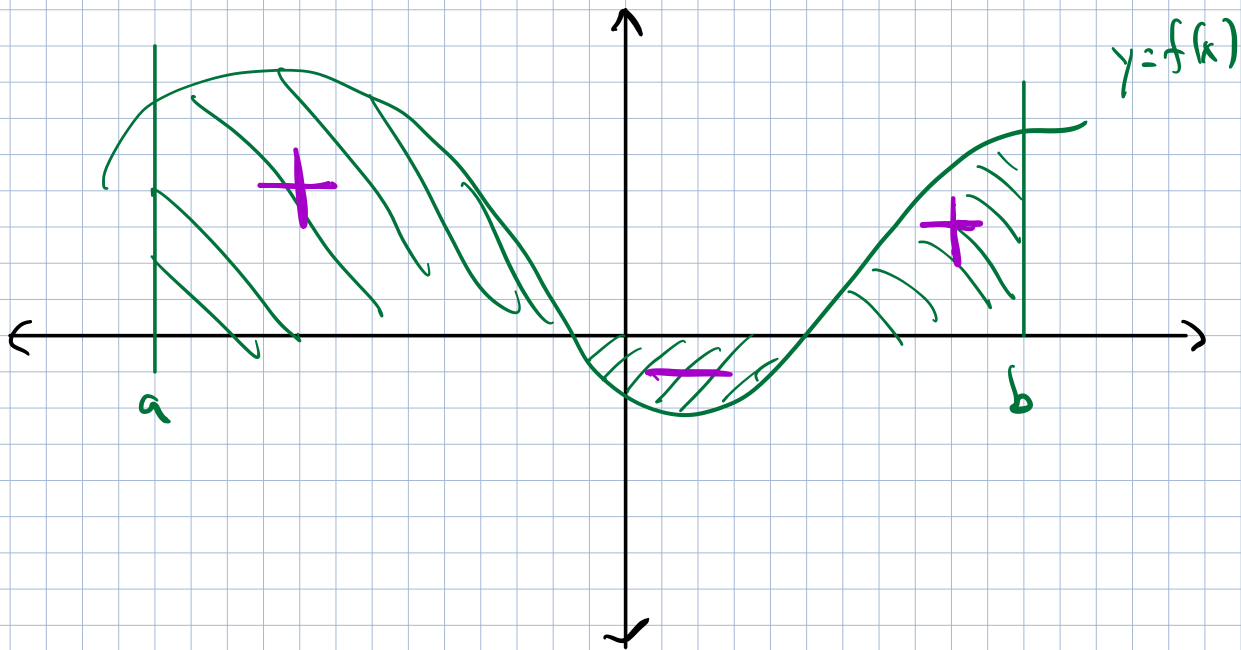
$$\int_a^b f(x)\,dx := \lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{\underbrace{f(a+i\cdot\Delta x)}_{\text{height of the } i\text{th rectangle}} \underbrace{\Delta x}_{\text{width of rectangle}}}_{\text{area of the } i\text{th rectangle}}$$

add up the areas of all n rectangles

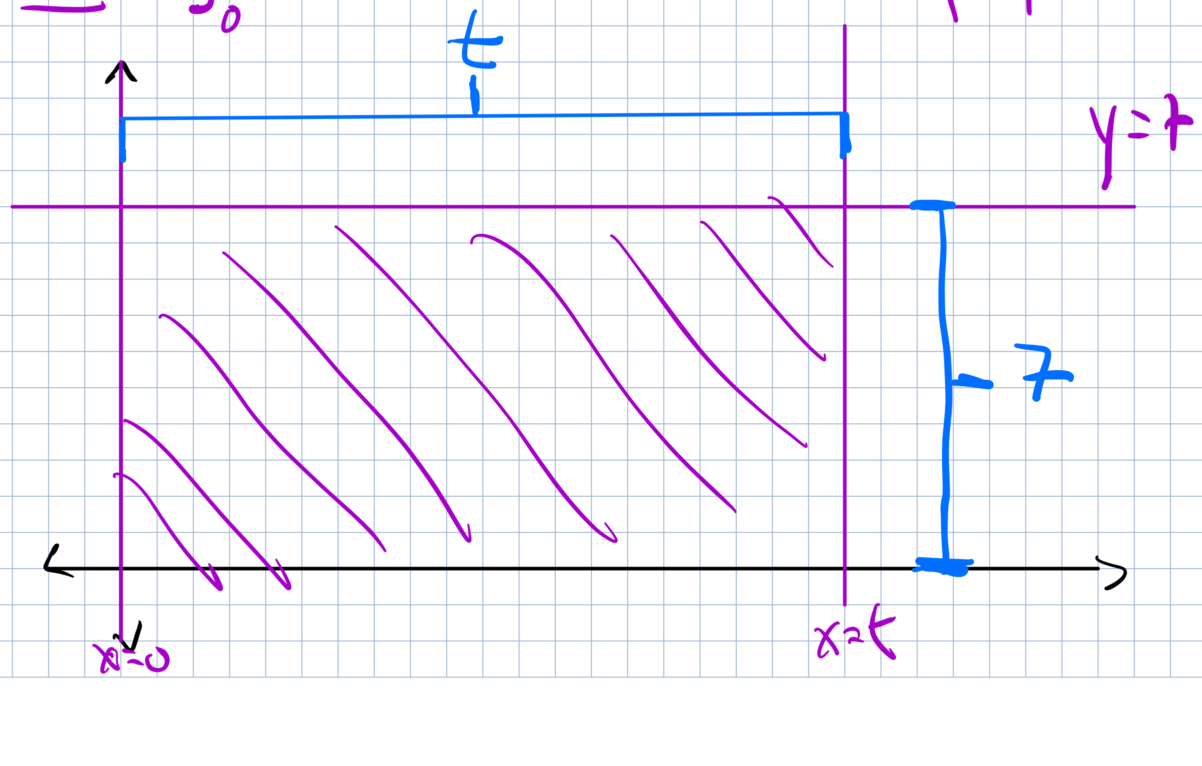
take the limit as the number of rectangles gets larger and larger

What does it really mean?

$\int_a^b f(x)\,dx$ is the signed area under the graph $y = f(x)$ between $x = a$ and $x = b$.

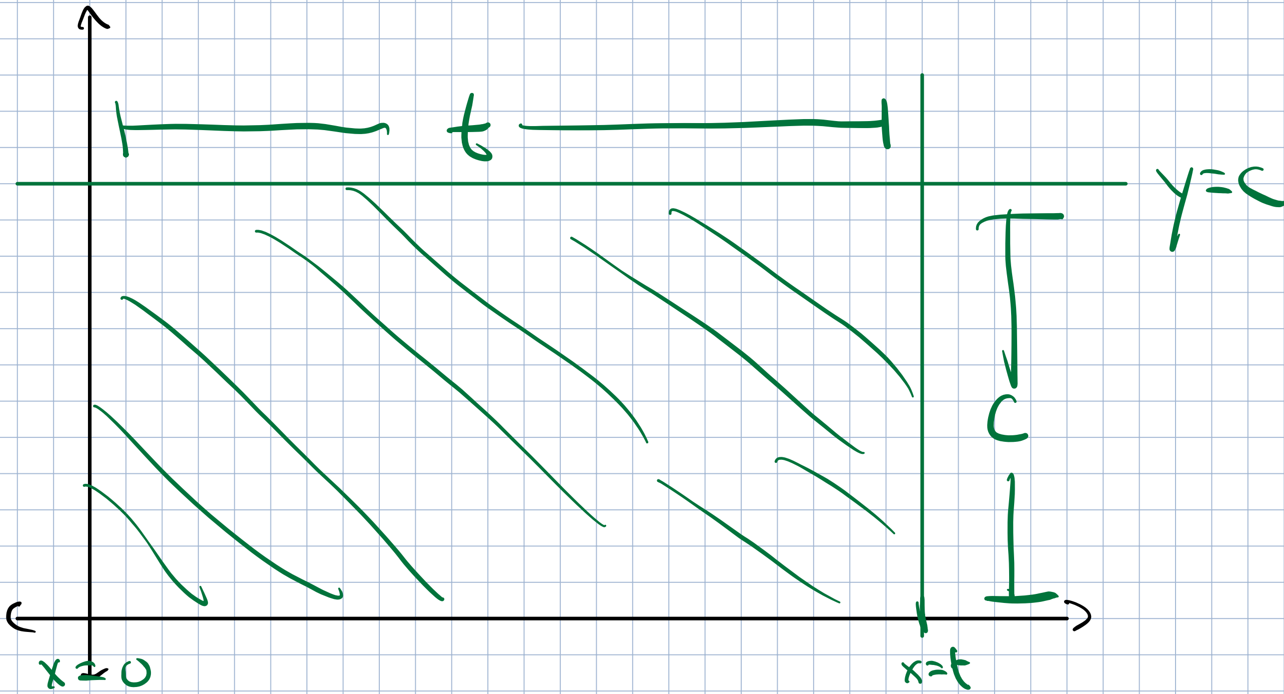


Ex: $\int_0^t 7\,dx$ = area of the purple rectangle



$$\int_0^t 7\,dx = 7\cdot t$$

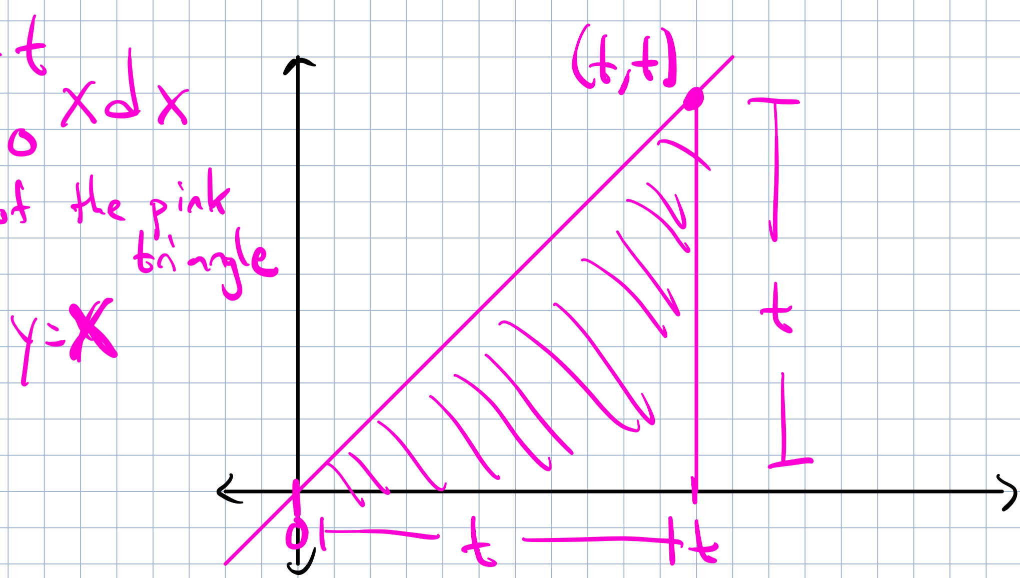
Ex: Let c be a constant. Compute $\int_0^t c\,dx$



$$\int_0^t c\,dx = \text{area of the green rectangle} = ct.$$

Ex: $\int_0^t x\,dx$

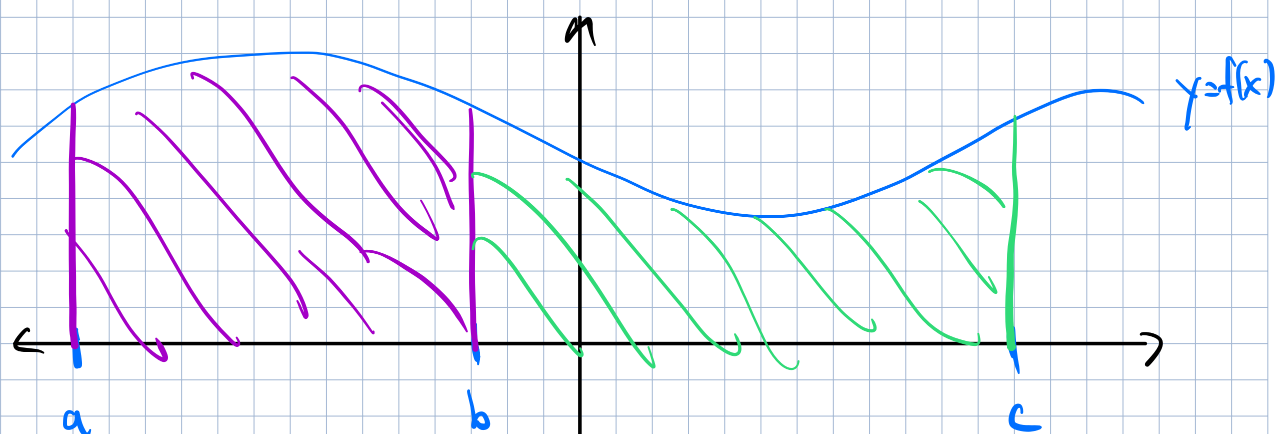
= area of the pink triangle



$$\int_0^t x\,dx = \frac{1}{2}\cdot t\cdot t = \frac{1}{2}t^2.$$

Properties of the Definite Integral

1) $\underbrace{\int_a^b f(x)\,dx} + \underbrace{\int_b^c f(x)\,dx} = \int_a^c f(x)\,dx$



2) $\int_a^a f(x)\,dx = 0.$

why?
by property (1)

$$\int_a^a f(x)\,dx + \cancel{\int_a^b f(x)\,dx} = \cancel{\int_a^b f(x)\,dx}$$

$$\int_a^a f(x)\,dx = 0$$

3) $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

Why?

by property (1),

by property (2)

$$\int_a^b f(x)\,dx + \int_b^a f(x)\,dx = \int_a^a f(x)\,dx = 0$$

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

4) $\int_a^b [f(x)+g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

5) If c is a constant, then

$$\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$$

Ex: Compute $\int_2^6 x\,dx$

$$\underbrace{\int_0^2 x\,dx} + \int_2^6 x\,dx = \underbrace{\int_0^6 x\,dx}$$

$$\frac{1}{2}\cdot 2^2 + \int_2^6 x\,dx = \frac{1}{2}\cdot 6^2$$

$$2 + \int_2^6 x\,dx = 18$$

$\int_2^6 x\,dx = 18 - 2 = 16.$

Ex: Compute $\int_a^b x\,dx$.

$$\underbrace{\int_0^a x\,dx} + \int_a^b x\,dx = \underbrace{\int_0^b x\,dx}$$

$$\frac{1}{2}\cdot a^2 + \int_a^b x\,dx = \frac{1}{2}b^2$$

$$\int_a^b x\,dx = \frac{1}{2}b^2 - \frac{1}{2}a^2.$$

Ex: $\int_0^6 (5x - 4)\,dx$

$= \int_0^6 5x\,dx + \int_0^6 (-4)\,dx$ sum rule (property (4))

$= 5\int_0^6 x\,dx - \int_0^6 4\,dx$ constant multiple rule (property (5))

$= 5 \cdot \frac{1}{2} \cdot 6^2 - 4 \cdot 6$

$= 90 - 24 = 66$

Ex: $\int_0^t x^2\,dx$.

We don't yet know how to do this without using the formal definition of the definite integral!

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(a+i\Delta x)\,\Delta x$$

where $\Delta x = \frac{b-a}{n}$.

$$\Delta x = \frac{t-0}{n} = \frac{t}{n}$$

$a=0$
$b=t$
$f(x)=x^2$

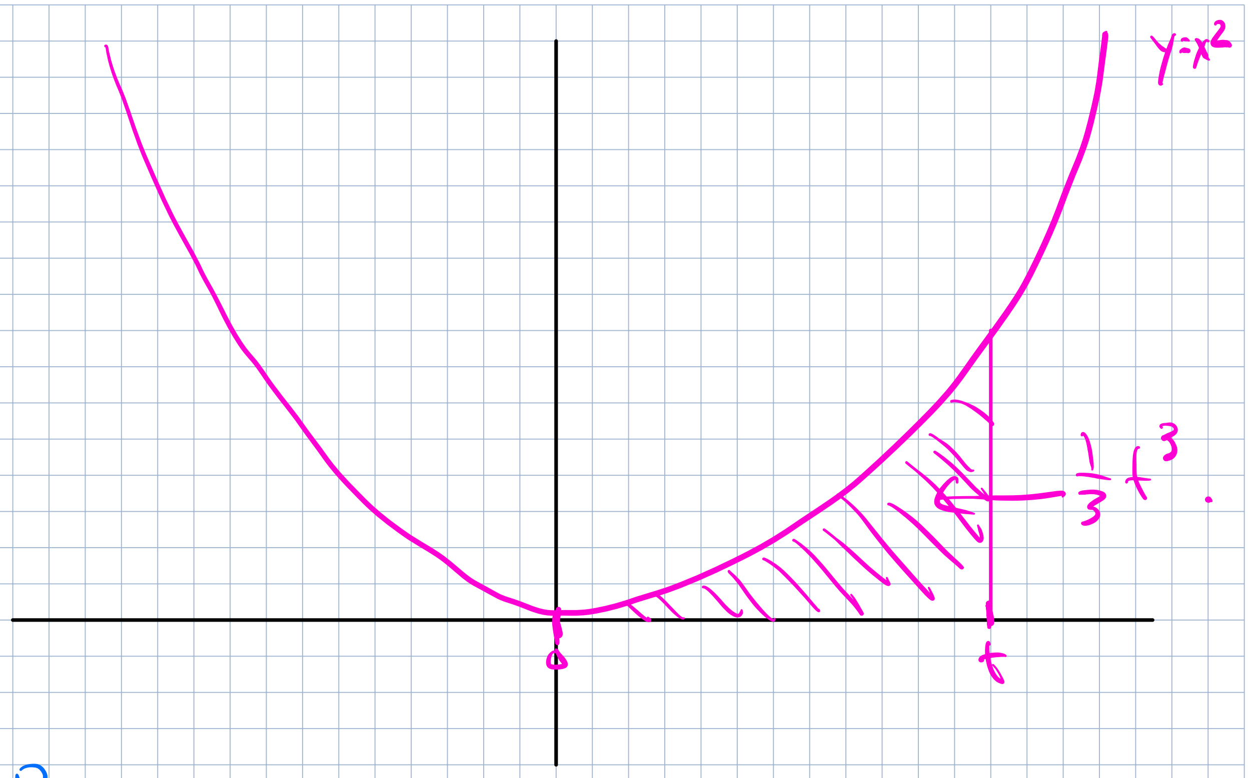
$$\int_0^t x^2\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left(i\frac{t}{n}\right)^2 \cdot \frac{t}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{t^3}{n^3}\, i^2$$

$$= \lim_{n\to\infty} \frac{t^3}{n^3} \sum_{i=1}^{n} i^2$$

$$= \lim_{n\to\infty} \frac{t^3}{n^3} \frac{n(n+1)(2n+1)}{6}$$

$$= \lim_{n\to\infty} \frac{t^3 n(n+1)(2n+1)}{6n^3} = \frac{2t^3}{6} = \frac{1}{3}t^3$$



Recap:

$$\int_0^t 1\,dx = t$$

$$\int_0^t x\,dx = \frac{1}{2}t^2$$

$$\int_0^t x^2\,dx = \frac{1}{3}t^3$$