Final Exam on Wednesday, May 4th 10:30 AM – 12:30 PM
here (Chem – Phys 139)
9 multiple choice
3 short answers

Course Evaluations are now Open!

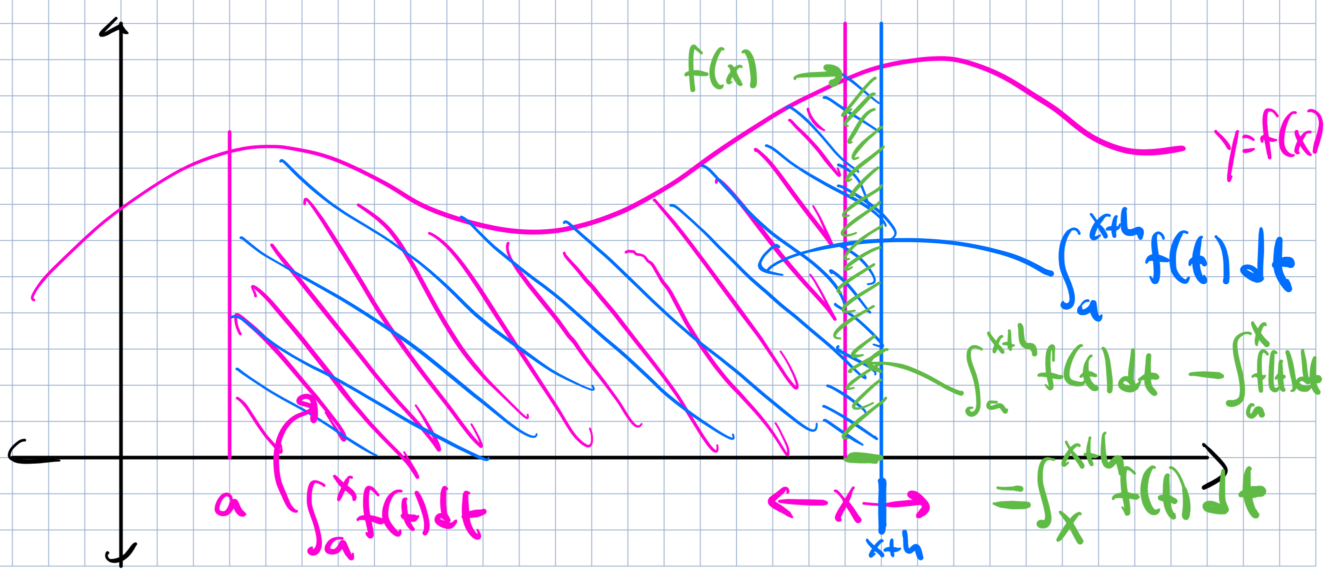
The Fundamental Theorem of Calculus

Last time:

$\int_0^x c\,dt = c\cdot x$	$\frac{d}{dx}[c\cdot x] = c$
$\int_0^x t\cdot dt = \frac{1}{2}x^2$	$\frac{d}{dx}\left[\frac{1}{2}x^2\right] = x$
$\int_0^x t^2\,dt = \frac{1}{3}x^3$	$\frac{d}{dx}\left[\frac{1}{3}x^3\right] = x^2$

Let $f(x)$ be a continuous function.

Consider the function $G(x) = \int_a^x f(t)\,dt$.



$\boxed{\begin{array}{l}\text{FTOC, Part 1: Let } f(x) \text{ be a continuous function,} \\ \text{and let } G(x) = \int_a^x f(t)\,dt. \\ \text{Then } G(x) \text{ is an antiderivative of } f(x). \\ \text{In other words, } G'(x) = f(x).\end{array}}$

Why is this true?

$$G'(x) = \lim_{h \to 0} \frac{G(x+h) - G(x)}{h}$$

$$= \lim_{h \to 0} \frac{\int_a^{x+h} f(t)\,dt - \int_a^x f(t)\,dt}{h}$$

$$= \lim_{h \to 0} \frac{\int_x^{x+h} f(t)\,dt}{h}$$

$$\approx \lim_{h \to 0} \frac{\left[\text{area of a rectangle w/ width } h \text{ and height } f(x)\right]}{h}$$

$$= \lim_{h \to 0} \frac{h f(x)}{h} = \lim_{h \to 0} f(x) = f(x)$$

Therefore, $G(x)$ is an antiderivative of $f(x)$.

FTOC, Part 2: Let $f(x)$ be a continuous function, and let $H(x)$ be <u>any</u> antiderivative of $f(x)$. Then $\int_a^b f(x)\,dx = H(b) - H(a)$.

Why is this true?

By FTOC, part 1, we saw $G(x)$ is an antiderivative of $f(x)$.

So $G(x)$ and $H(x)$ differ by a constant.

In other words, $H(x) = G(x) + C$.

$$H(b) - H(a) = [G(b) + C] - [G(a) + C]$$

$$= G(b) - G(a) = \int_a^b f(t)\,dt - \cancel{\int_a^a f(t)\,dt}^{\,0}$$

$$= \int_a^b f(t)\,dt$$

Ex: $\int_0^1 x^2\,dx$

$H(x) = \frac{1}{3}x^3$ is an antiderivative of x^2.

$$\int_0^1 x^2\,dx = \frac{1}{3}x^3\Big|_0^1 = \frac{1}{3}\cdot 1^3 - \frac{1}{3}\cdot 0^3$$

$$= \frac{1}{3}$$

Ex: Compute $\int_2^5 [3x^3 - 5x^2 + 4]\,dx$

I need an antiderivative of $3x^3 - 5x^2 + 4$.

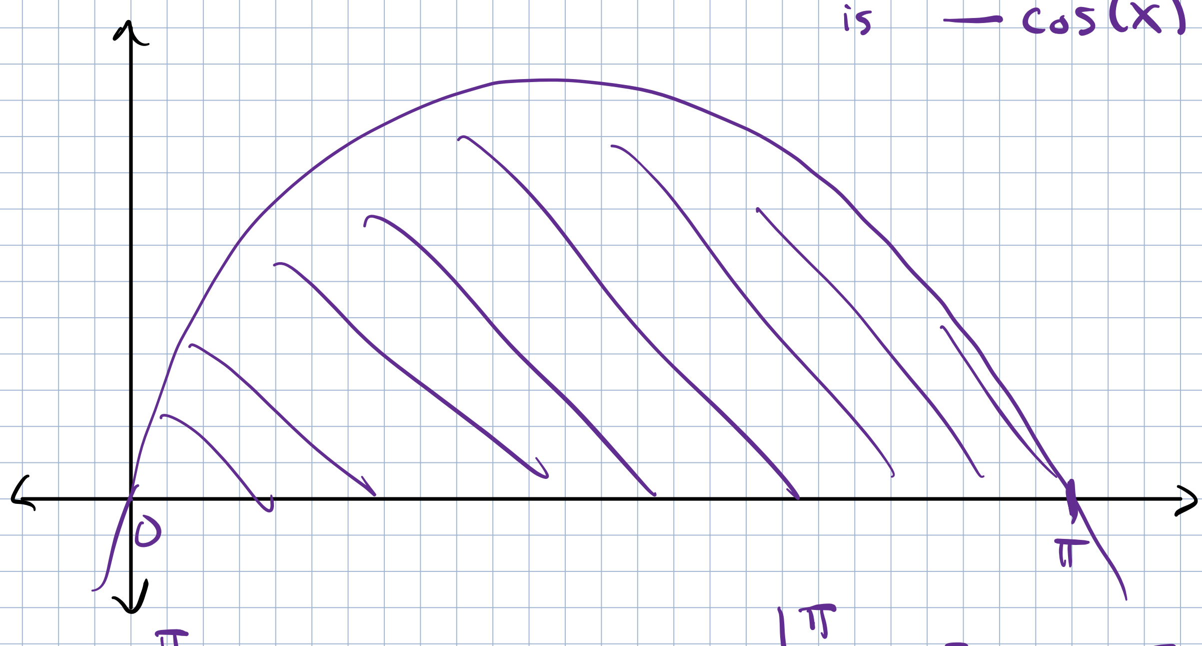
$$\int_2^5 [3x^3 - 5x^2 + 4]\,dx = 3\int_2^5 x^3\,dx - 5\int_2^5 x^2\,dx + \int_2^5 4\,dx$$

$$= \left[3 \cdot \frac{1}{4} x^4 - 5 \cdot \frac{1}{3} x^3 + 4x\right]\Big|_2^5$$

$$= \left[\frac{3}{4} \cdot 5^4 - \frac{5}{3} \cdot 5^3 + 4 \cdot 5\right] - \left[\frac{3}{4} \cdot 2^4 - \frac{5}{3} \cdot 2^3 + 4 \cdot 2\right]$$

Ex: $\int_0^\pi \sin(x)\,dx$

An antiderivative of $\sin(x)$ is $-\cos(x)$.

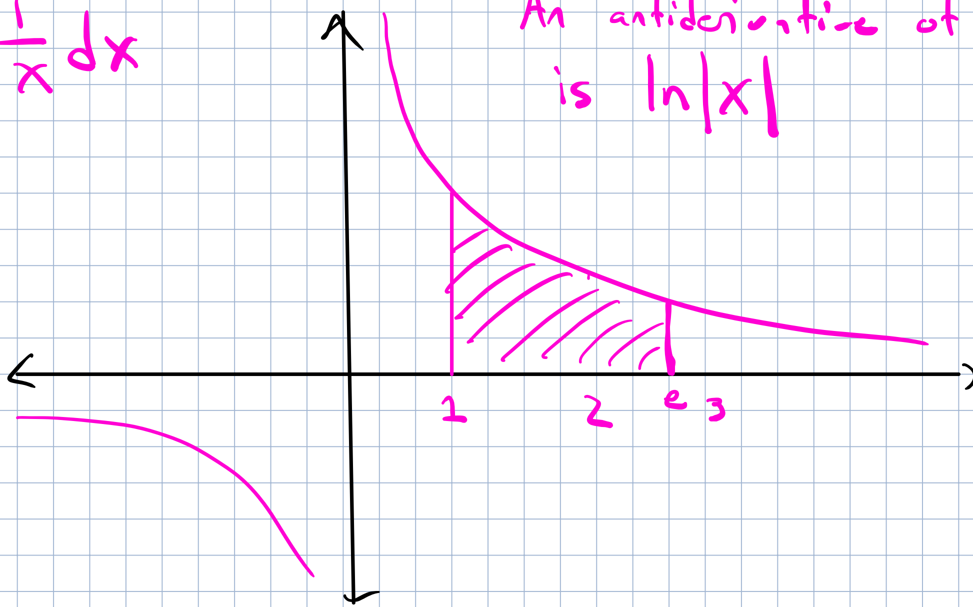


$$\int_0^\pi \sin(x)\,dx = -\cos(x)\Big|_0^\pi = [-\cos(\pi)] - [-\cos(0)]$$

$= -(-1) - (-1) = 1 + 1 = 2$

Ex: $\int_1^e \frac{1}{x}\,dx$

An antiderivative of $\frac{1}{x}$ is $\ln|x|$



$$\int_1^e \frac{1}{x}\,dx = \ln|x|\Big|_1^e = \ln(e) - \ln(1)$$

$$= 1 - 0 = 1$$