First exam IN CLASS February 2^{nd}

Please send me your DRC letters

Exponential Growth

Ex: Population of skunks.

Start with 2 skunks at time $t = 1$.

Population doubles every month.

Time (months)	1	2	3	4	5	6	...
Population	2	4	8	16	32	64	

$N(t)$ = skunk population at time t.

$N(t) = 2^t, \quad t = 1, 2, 3, 4, \ldots$

Discrete

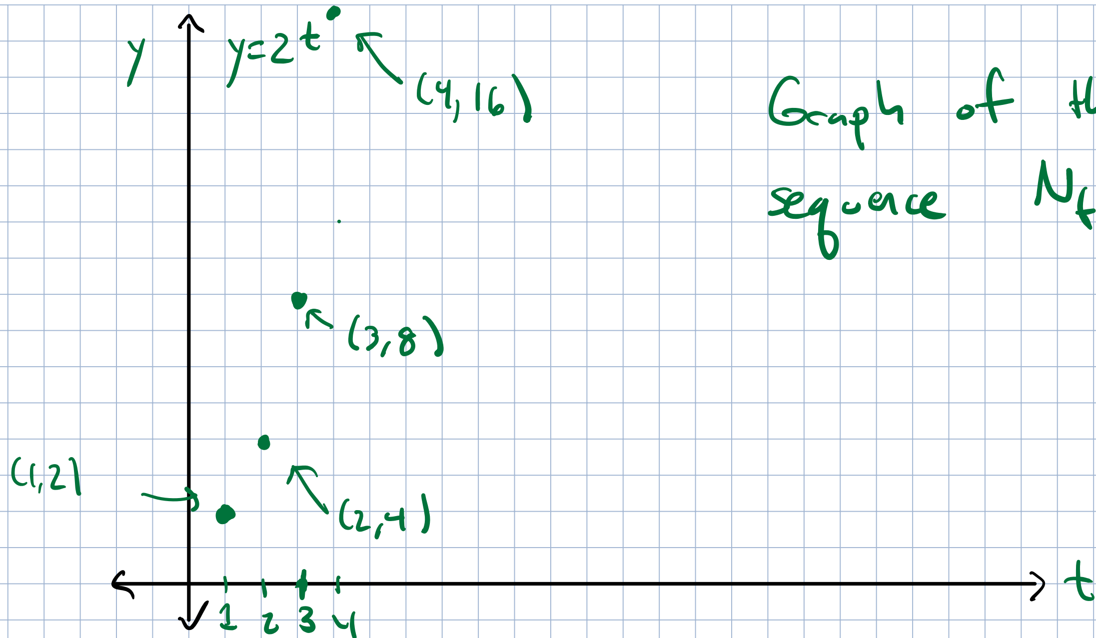
$N(t)$ is what we call a sequence.

A sequence is a function that assigns a number to every nonnegative whole number.

$N_0, N_1, N_2, N_3, \ldots$

Ex:

Graph of the sequence $N_t = 2^t$.



Ex: Starting population at time $t=0$ is 100 skunks.
Population doubles every month.

t	0	1	2	3	4	5	...
N_t	100	200	400	800	1600	3200	...

$$N_t = 100 \cdot 2^t$$

Ex: Starting population at time $t=0$ is 100 skunks.
Population triples every month.

t	0	1	2	3	4	...
N_t	100	300	900	2700	8100	...

$$N_t = 100 \cdot 3^t$$

In general, if the starting population is N_0 and the population increases by a factor of R each month, then...

t	0	1	2	3	4	...
N_t	N_0	$N_0 \cdot R$	$N_0 \cdot R^2$	$N_0 \cdot R^3$	$N_0 \cdot R^4$	...

$$\boxed{N_t = N_0 \cdot R^t.}$$ ← Exponential Growth

Exponential Decay

If $0 < R < 1$, then $N_t = N_0 \cdot R^t$ is a decreasing sequence. In this case, we refer to it as exponential decay.

Ex: If $N_0 = 1$, $R = \frac{1}{2}$.

t	0	1	2	3	4	...
N_t	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	...

$$N_t = 1 \cdot \left(\frac{1}{2}\right)^t = \frac{1}{2^t}.$$

Recursively Defined Sequences

A recursively defined sequence is an expression for N_{t+1} in terms of N_t.

Ex: $N_{t+1} = 2 \cdot N_t.$ ← every time the month increases by 1, the population doubles.

$N_0 = 1.$

$N_1 = 2 \cdot N_0 = 2 \cdot 1 = 2$

$N_2 = 2 \cdot N_1 = 2 \cdot 2 = 4$

$N_3 = 2 \cdot N_2 = 2 \cdot 4 = 8$

$N_0 = 100$

$N_1 = 2 \cdot N_0 = 2 \cdot 100 = 200$

$N_2 = 2 \cdot N_1 = 2 \cdot 200 = 400$

$N_3 = 2 \cdot N_2 = 2 \cdot 400 = 800$

Ex: $N_{t+1} = 3 \cdot N_t$ ← every term in the sequence is 3 times the previous term.

$N_0 = 100$

$N_1 = 3 \cdot N_0 = 3 \cdot 100 = 300$

$N_2 = 3 \cdot N_1 = 3 \cdot 300 = 900$

$N_3 = 3 \cdot N_2 = 3 \cdot 900 = 2700.$

In general, consider the recursion

$N_{t+1} = N_t \cdot R$ ← each term in the sequence is R times the previous term.

$N_0 = N_0 .$

$N_1 = N_0 \cdot R$

$N_2 = N_1 \cdot R = (N_0 \cdot R) \cdot R = N_0 \cdot R^2$

$N_3 = N_2 \cdot R = (N_0 \cdot R^2) \cdot R = N_0 \cdot R^3$

$N_4 = N_3 \cdot R = (N_0 \cdot R^3) \cdot R = N_0 \cdot R^4$

The sequence is $N_t = N_0 \cdot R^t .$