Exam 1 IN CLASS Wednesday, Feb 2

If you feel sick, DO NOT COME

Email me instead at dave.h.jensen@gmail.com

(NOT through Canvas)

Final Project due April 18th

Population Models

Ex: You have a population of 100 skunks. The number of skunks doubles every year.

Find N_t = # of skunks after t years.

N_0 = initial population $= 100$.

$$R = \frac{N_{t+1}}{N_t} = 2.$$

$$N_t = N_0 \cdot R^t$$

$$= 100 \cdot 2^t$$

This is a fairly good model, but in the long run it's unrealistic because

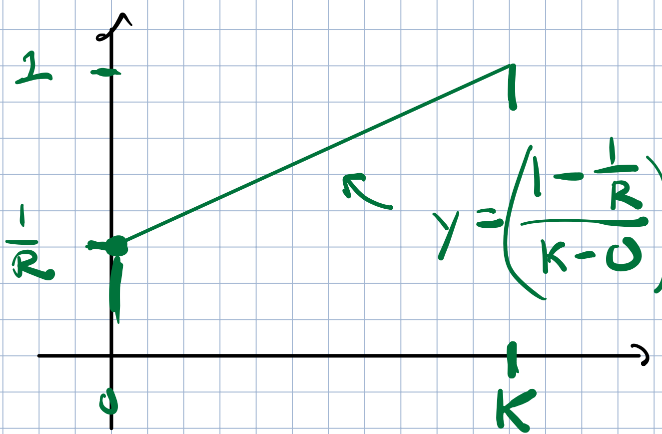
$$\lim_{t\to\infty} N_t = \infty.$$

Beverton-Holton Recruitment Curve

$\frac{1}{R} = \frac{N_t}{N_{t+1}}$ is a constant in the previous example.

Instead, assume $\frac{N_t}{N_{t+1}}$ is a <u>linear</u> function.

going through the points $(0, \frac{1}{R})$, $(K, 1)$.



$$y = \left(\frac{1-\frac{1}{R}}{K-0}\right) N_t + \frac{1}{R}$$

$y = \frac{N_t}{N_{t+1}}$

$x = N_t$

$$\frac{N_t}{N_{t+1}} = \frac{1-\frac{1}{R}}{K} N_t + \frac{1}{R}.$$

$$\frac{1}{N_{t+1}} = \frac{1-\frac{1}{R}}{K} + \frac{1}{RN_t}$$

$$= \frac{RN_t - N_t}{RKN_t} + \frac{K}{RKN_t}$$

$$= \frac{RN_t - N_t + K}{RKN_t}$$

$$N_{t+1} = \frac{RKN_t}{RN_t - N_t + K} = \frac{RKN_t}{(R-1)N_t + K}$$

$$\boxed{N_{t+1} = \frac{RN_t}{1 + \frac{R-1}{K} \cdot N_t}}$$

Recursion for the BHRC.

Limiting Behavior of N_t.

Find the fixed points. Plug in $N_t = N_{t+1} = a$.

$$a = \frac{Ra}{1 + \frac{R-1}{K} a}$$

Solve for a.

$$a\left(1 + \frac{R-1}{K} a\right) = Ra$$

$$a\left(1 + \frac{R-1}{K} a\right) - Ra = 0$$

$$a\left(1 - R + \frac{R-1}{K} a\right) = 0$$

Either $a = 0$ or

$$(1-R) + \frac{R-1}{K} a = 0$$

$$-1 + \frac{1}{K} a = 0$$

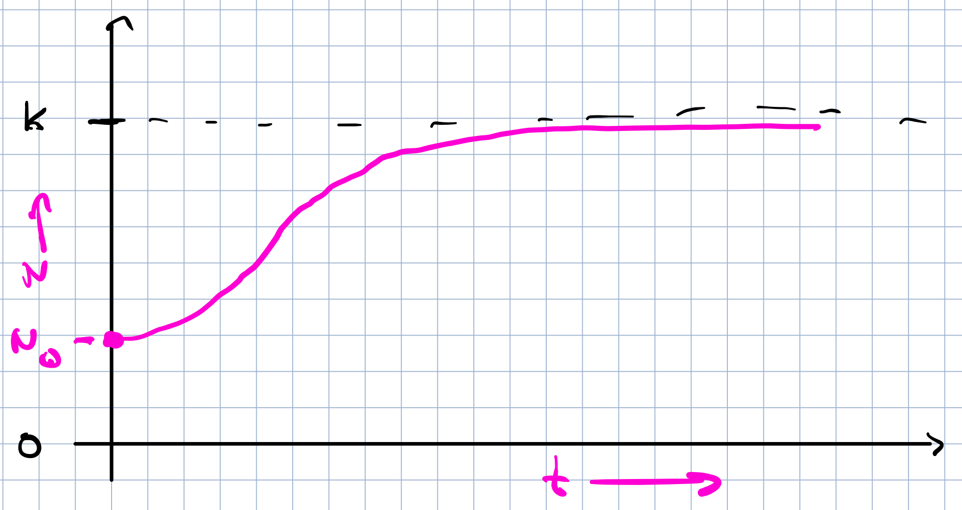
$$\frac{1}{K} a = 1$$

$$a = K.$$

Two fixed points are: $a = 0, K$.

The fixed point $a=0$ is locally unstable.
The fixed point $a=K$ is locally stable.

If $N_0 > 0$, then $\lim_{t\to\infty} N_t = K$.



Discrete Logistic Equation

$$\boxed{N_{t+1} = N_t \cdot \left[1 + R\left(1 - \frac{N_t}{K}\right)\right]}$$

Describe the limiting behavior.

Find the fixed points. Plug in $a = N_t = N_{t+1}$

$$a = a \cdot \left[1 + R\left(1 - \frac{a}{K}\right)\right].$$

Solve for a.

$$0 = a \cdot \left[1 + R\left(1 - \frac{a}{K}\right)\right] - a \quad = a\left[\cancel{1} + R\left(1 - \frac{a}{K}\right) \cancel{-1}\right]$$

$$= a \cdot \left[R\left(1 - \frac{a}{K}\right)\right]$$

$$0 = a \cdot \left(1 - \frac{a}{K}\right)$$

Either $a = 0$ or $1 - \frac{a}{K} = 0.$

$$1 = \frac{a}{K}.$$

$$K = a.$$

Fixed points are: $a = 0, K$.

$a = 0$ is a locally unstable fixed point.

$a = K$ is a locally stable fixed point.

If $N_0 > 0$, then $\lim_{t \to \infty} N_t = K$.

<u>Ricker's Curve:</u>

$$N_{t+1} = N_t \cdot e^{R(1 - N_t/K)}.$$

$$\exp(x) = e^x.$$

$$\boxed{N_{t+1} = N_t \cdot \exp\left[R\left(1 - N_t/K\right)\right].}$$

Find the limiting behavior.

Find the fixed points.

$$a = a \cdot \exp\left[R\left(1 - \cancel{N_t}\, a/K\right)\right]$$

$$0 = a \cdot \left(\exp\left[R\left(1 - \frac{\cancel{N_t}\, a}{K}\right)\right] - 1\right)$$

Either $a = 0$ or

$$\exp\left[R\left(1 - \frac{\cancel{N_t}\, a}{K}\right)\right] - 1 = 0$$

$$\exp\left[R\left(1 - \frac{a}{K}\right)\right] = 1$$ Take ln of both sides

$$R\left(1 - \frac{a}{K}\right) = \ln(1) = 0$$

$$1 - \frac{a}{K} = 0$$

$$1 = \frac{a}{K} \qquad K = a.$$

Fixed points are $a = 0, K$.

Again, $a = 0$ is locally unstable and $a = K$ is locally stable, so if $N_0 > 0$, then $\lim_{t \to \infty} N_t = K$.