Exam 1 Wednesday IN CLASS

If you're sick <u>STAY</u> <u>HOME</u>, email me at dave.h.jensen@gmail.com (do <u>NOT</u> use Canvas)

Exam Format

50 minutes

6 multiple choice

2 short answers → SHOW YOUR WORK

Calculators OK

What's on the Exam?

- Elementary Functions
  - linear fns, quadratic fns, composition and inverses, graphing
- Exponentials
  - definition, properties, graph
- Logarithms
  - definition, properties, graph
- Sequences
  - exponential, recursive sequences
- Limits of Sequences
  - definition, how to find limits of recursive sequences

↳ • Semi-log and Double Log Plots

- graphs in these coordinates, translating between log coordinates and standard coordinates

Semilog Plots

Graph a fn $y = f(x)$ where the vertical axis is $Y = \log(y)$.

If the graph of a function in a semilog plot is a line, then the function is an exponential function.

The graph of $\underline{y = a \cdot b^x}$ in a semilog plot is a line w/ slope $\log(b)$ and y-intercept $\log(a)$.

$$Y = \log(y) = \log(a \cdot b^x)$$
$$= \log(a) + \log(b^x)$$
$$= \log(a) + \log(b) \cdot x$$

Ex: Consider the function $\underline{y = 10 \cdot (1000)^x}$.

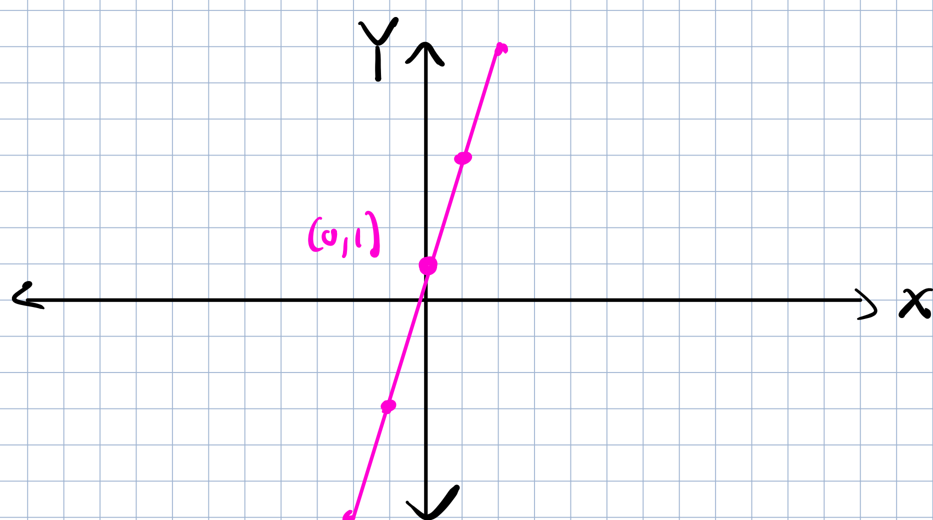
a) Let $Y = \log(y)$ and write Y as a function of x.

b) Graph this function on a semilog plot.

a) $y = \underline{a \cdot b}^x$ $\quad a = 10$, $b = 1000$

$$Y = \log(10) + \log(1000) \cdot X$$
$$= 1 + 3 \cdot X$$

b)



$\underline{\text{Ex}}$: A double-log plot is where you graph a function $Y = f(x)$ where the vertical axis is $Y = \log(y)$ and the horizontal axis is $X = \log(x)$.

If the graph of a function on a double-log plot is a line, then that function is a power function: $y = a \cdot x^m$.

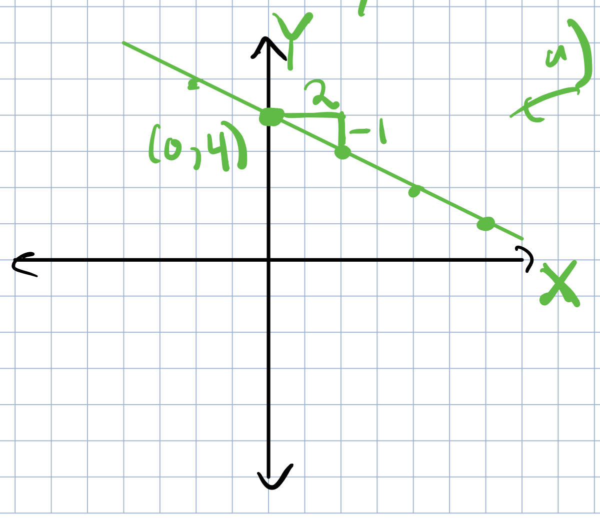
Specifically, the graph of $y = a \cdot x^m$ on a double-log plot is a line with slope m and y-intercept $\log(a)$.

$$y = a \cdot x^m$$

$$Y = \log(y) = \log(a \cdot x^m)$$
$$= \log(a) + \log(x^m)$$
$$= \log(a) + m \cdot \log(x)$$
$$= \log(a) + m \cdot X$$

Ex: Suppose I give you this double-log plot.

a) Write Y as a fn in terms of X.
b) Write y as a fn in terms of x.
c) Graph y as a fn in terms of x.



a) linear fn with y-intercept 4 and slope $-\frac{1}{2}$

$$Y = 4 - \frac{1}{2} X.$$

b) We want to turn this into a power function of the form $y = a \cdot x^m$

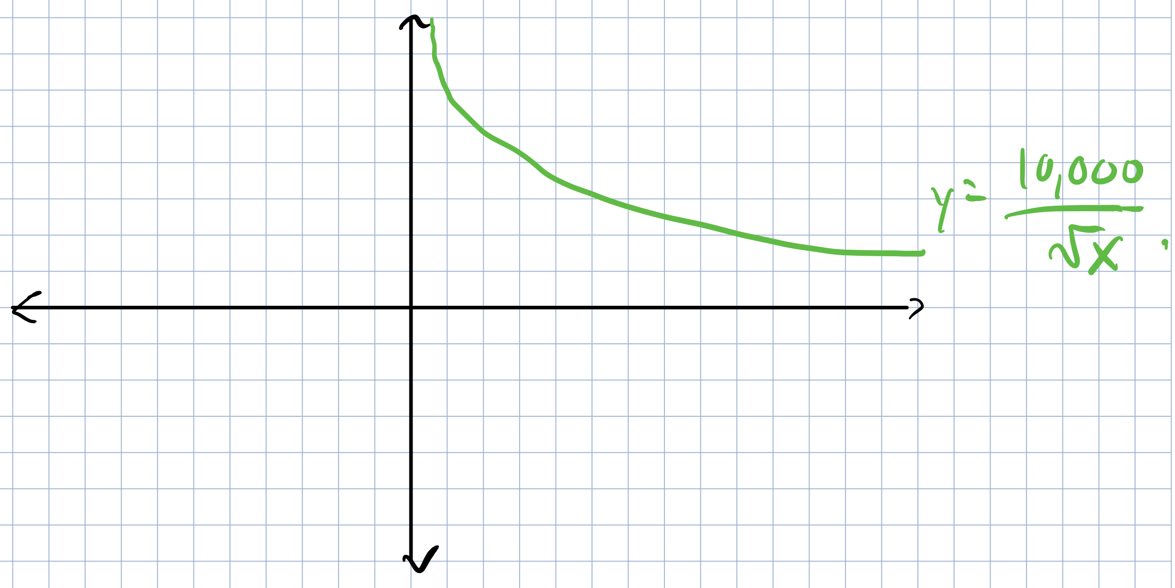
y-intercept: $Y = \log(a)$ ←

$10000 = 10^4 = 10^{\log(a)} = a$

slope: $m = -\frac{1}{2}$

$$y = 10{,}000 \cdot x^{-1/2} = \frac{10{,}000}{\sqrt{x}}$$

c)



Limits of Sequences

Ex: Consider the recursive sequence:

$$a_{n+1} = \frac{6}{a_n + 1}, \quad a_0 = 0.$$

Find $\lim_{n \to \infty} a_n$.

First, find the fixed points.
Plug in $a = a_n = a_{n+1}$.

$$a = \frac{6}{a+1}$$ Solve for a.

$$a(a+1) = 6$$
$$a^2 + a = 6$$
$$a^2 + a - 6 = 0$$

Q.F.: $a = \frac{-1 \pm \sqrt{1^2 - 4(-6)}}{2 \cdot 1}$

$$a = \frac{-1 \pm \sqrt{1+24}}{2} = \frac{-1 \pm \sqrt{25}}{2}$$

$$= \frac{-1 \pm 5}{2}$$

$a = -3, 2 \leftarrow$ Fixed points.

$$a_{n+1} = \frac{6}{a_n + 1} \qquad a_0 = 0.$$

$$a_0 = 0$$
$$a_1 = \frac{6}{a_0+1} = \frac{6}{0+1} = 6$$
$$a_2 = \frac{6}{a_1+1} = \frac{6}{6+1} = \frac{6}{7}$$
$$a_3 = \frac{6}{a_2+1} = \frac{6}{\frac{6}{7}+1} = \frac{6}{\frac{13}{7}} = \frac{42}{13}$$

These numbers are all positive, so they can't be approaching -3.

So $\lim_{n\to\infty} a_n = 2$.

Alternative approach: plug in values close to the fixed points.

For $a=2$, plug in 2.001

$$a_{n+1} = \frac{6}{2.001+1} = 1.999334\ldots$$

Closer to 2, so this is a locally stable fixed pt.

For $a=-3$, plug in -3.001

$$a_{n+1} = \frac{6}{-3.001+1} = -2.99850\ldots$$

Further from -3, so this is a locally unstable fixed point.

Ex: $a_{n+1} = \dfrac{6}{a_n+1}$, $a_0 = -3$.

$a_0 = -3$

$$a_1 = \frac{6}{-3+1} = \frac{6}{-2} = -3$$

$$a_2 = \frac{6}{a_1 + 1} = \frac{6}{-3+1} = -3$$

Sequence is $-3, -3, -3, -3, \ldots$

Limit is -3.