grandson of Genghis Khan. The caliphate at Baghdad was destroyed in 1258, and a century later a new empire was created under Timur (also known as Tamburlaine or Timur the Lame). But mathematical activities continued throughout this period, probably because of the patronage offered by Hulegu Khan to astronomers of the caliber of al-Tusi, and by Ulugh Beg, grandson of Timur, to a later group that included al-Kashi.[18]

Little is known of al-Kashi's life until 1406, when he began a series of observations of lunar eclipses from his birthplace, Kashan. At Samarkand, Ulugh Beg had established an observatory and a madrassa (a school of advanced study in science or theology), and it is probable that al-Kashi was invited to join a group of scientists there. We know that in 1414 he revised a set of astronomical tables produced by al-Tusi and dedicated it to Ulugh Beg, who was a knowledgeable astronomer himself. Samarkand under the rule of Ulugh Beg had become an intellectual center where, as al-Kashi observed in a letter to his father, "the learned are gathered together, and teachers who hold classes in all the sciences are at hand, and the students are all at work on the art of mathematics."

AI-Kashi's strength lay in prodigious calculations. His approximation for π, correct to sixteen decimal places, was obtained by circumscribing a circle by a polygon having 3×2^{28} (805,306,368) sides. His best-known work, *Miftah al-hisab* (The Key of Arithmetic), completed in 1427, provides us with a compendium of the best of Islamic arithmetic and algebra. Its contents include the first systematic exposition of decimal fractions; a method of extracting the *n*th root of a number, similar to the so-called Horner's method and thus probably derived from the Chinese; and the solution of a cubic equation to obtain a value for the sine of one degree. On al-Kashi's death in 1429, Ulugh Beg praised the mathematician's achievements, in the preface to his own *zij*, calling him "the admirable mullah known among the famous of the world, who had mastered and completed the science of the ancients, and who could solve the most difficult problems."

Medieval Islam's Role in the Rise and Spread of Indian Numerals

In chapter 8 we saw how the Indian numerals evolved and spread to Southeast Asia; we now take up the story of their spread westward. The Islamic world contained the leading actors in this drama. The first evidence of the westward migration of Indian numerals is found in the following (rather aggrieved) passage from the fragments of a book in Syriac. It was written in 662 by a Nestorian bishop, Severus Sebokht, who came from Keneshra in the upper reaches of the Euphrates. He had written previously on both geography and astronomy, and hurt by the arrogance of some Greek (or Byzantine) scholars who looked down on his people, he wrote:

> I will omit all discussion of the science of the Indians, a people not the same as the Syrians; of their subtle discoveries in astronomy, discoveries that are more ingenious than those of the Greeks and the Babylonians; and of their valuable methods of calculation which surpass description. I wish only to say that this computation is done by means of nine signs. If those who believe, because they speak Greek, that they have arrived at the limits of science, [would read the earlier texts], they would perhaps be convinced, even if a little late in the day, that there are others also who know something of value.

This supports the view that, even before the beginning of Islamic rule, knowledge of Indian numerals had spread westward, probably as a result of widespread interest in Indian astronomy. Christian sects, particularly the Nestorian and Syrian Orthodox denominations, needed to calculate an accurate date for Easter, and various astronomical texts were examined with this problem in mind. (It was a problem that continued to occupy mathematicians, including Gauss, down to the nineteenth century.) There is also the possibility, given the thriving commercial relations between Alexandria and India, that the Indian numeral system had reached the shores of Egypt as early as the fifth century AD. It would have been regarded as a useful commercial device rather than a system that might become more widely used or accepted; it would not have been adopted for scientific and astronomical calculations by Alexandrian scientists, who used the Mesopotamian sexagesimal system.

After the Islamic conquest, Indian numerals probably arrived at Baghdad in 773 with the diplomatic mission from Sind to the court of al-Mansur. Around 825, al-Khwarizmi wrote his famous *Book of Addition and Subtraction according to the Indian Calculation*, the first text to deal with the new numerals. As mentioned earlier, although the original Arabic text is now lost, it gave rise to a whole genre of works in Arabic, Latin, and Greek. Europe came to know it only through several partial Latin translations undertaken five hundred years later by John of Seville and Robert of

Chester. The text contained a detailed exposition of both the representation of numbers and operations using Indian numerals. Al-Khwarizmi was at pains to point out the usefulness of a place-value system incorporating zero, particularly for writing large numbers.

The earliest extant texts to examine arithmetical operations with Indian numerals are Abul Hassan al-Uqlidisi's *Kitab al-fusul fil-hisab al-Hindi* (The Book of Chapters on Indian Arithmetic, 952) and Kushyar ibn Labban's *Usul hisab al-Hind* (Principles of Indian Reckoning, c. 1010). Operations are given in both the Indian decimal place-value system and the Babylonian sexagesimal system. Al-Uqlidisi's book is particularly notable for the first use of decimal fractions in computing with the new numerals; Ibn Labban introduces "Indian" reckoning as part of a general discussion of Indian methods used in astronomical calculations. Ibn Labban's work became an influential arithmetic textbook in the Islamic world. Some aspects of both works will be highlighted in the next section.

Other references to Indian numerals are found in works by later writers. The opinions of the tenth-century polymath Abu Rayhan al-Biruni are particularly valuable since he lived in India and knew Sanskrit. Two of his books, *Risalah* (Book of Numbers) and *Rasum al-Hind* (Indian Arithmetic) contain an assessment of Indian numeration as well as some corrections to earlier works on the subject.

A minor incident from the autobiography of Ibn Sina (c. 980–1037) shows us how the use of the Indian numerals was spreading. When he was about ten years old a group of missionaries belonging to a small Islamic sect came to Bukhara from Egypt, and it was from these people that Ibn Sina learned "Indian arithmetic." There is also a story of the young Ibn Sina being taught "Indian calculation" and algebra by a vegetable vendor. What these stories illustrate is that by the beginning of the eleventh century Indian numeration was being used from the borders of central Asia to the southern reaches of the Islamic empire in North Africa and Egypt—and not just by scholars.

In the transmission of Indian numerals to Europe, as with almost all knowledge obtained from the Islamic world, Spain and (to a lesser extent) Sicily played the role of intermediaries, being the two areas in Europe that had been under Islamic rule for many years. (This was one of the important aspects emphasized in our examination of the spread of mathematical knowledge in chapter 1.) So it is not surprising to find that the oldest record of Indian numeration in Europe, dating from the year 976, is found in a monastery in northern Spain. This manuscript, known as the *Codex Vigilanus*, is now kept in a museum in Madrid. The relevant passage reads:

> So with computing symbols. We must realize that the Indians had the most penetrating intellect, and other nations were way behind them in the art of computing, in geometry, and in other free [? probably meaning natural] sciences. And this is evident from the nine symbols with which they represented every rank of number at every level.

There follows a set of symbols, now known as the West Arabic or *Ghubar* (Gobar) numerals, from which our present numerals derive. The shapes of these numerals are shown in figure 11.2, which outlines the evolution of our numerals from some of the earlier forms. The Indian numerals from which the two main forms of Arabic numerals (East and West) were derived quite likely resembled those found in the Gwalior inscription of 876, which we discussed in chapter 8. The western version of the Arabic numerals that stemmed from Indian figures were called *Ghubar* numerals—presumably because, as the word *Ghubar* suggests, these symbols were written on a sand board containing dust, a practice that was popular in India. The *Ghubar* numerals were widely used in the western part of the Islamic empire, including Spain and Sicily; indeed, they are still found in parts of North Africa. The eastern Arabic numerals may have come to the Islamic world by a more indirect route that included Persia. In the early years the differences between the two types of Arabic numerals were slight, but they grew with the passage of time. A striking feature of the evolution of Indian numerals from the Gwalior script to the present form, as shown in figure 11.2, is how little they have changed on passing through one culture after another. In several instances what changed was the orientation of a symbol, not its form.

The oldest date to appear in the new numerals in Europe is on a Sicilian coin from the reign of the Norman king Roger II. On it the year is expressed as AH 533 (AD 1138). The use of the Muslim date is not surprising, since Roger II encouraged the pursuit of Islamic learning in his kingdom.

We now come to a landmark in the spread of Indian numerals: the appearance of one of the most influential mathematical texts in medieval Europe, the *Liber Abaci* (Book of the Abacus) by Fibonacci (1170–1250). The young Fibonacci grew up in North Africa, where his father was in charge of a customshouse. He was first introduced to Indian numerals by his Islamic teachers there. As a young man, he traveled extensively around

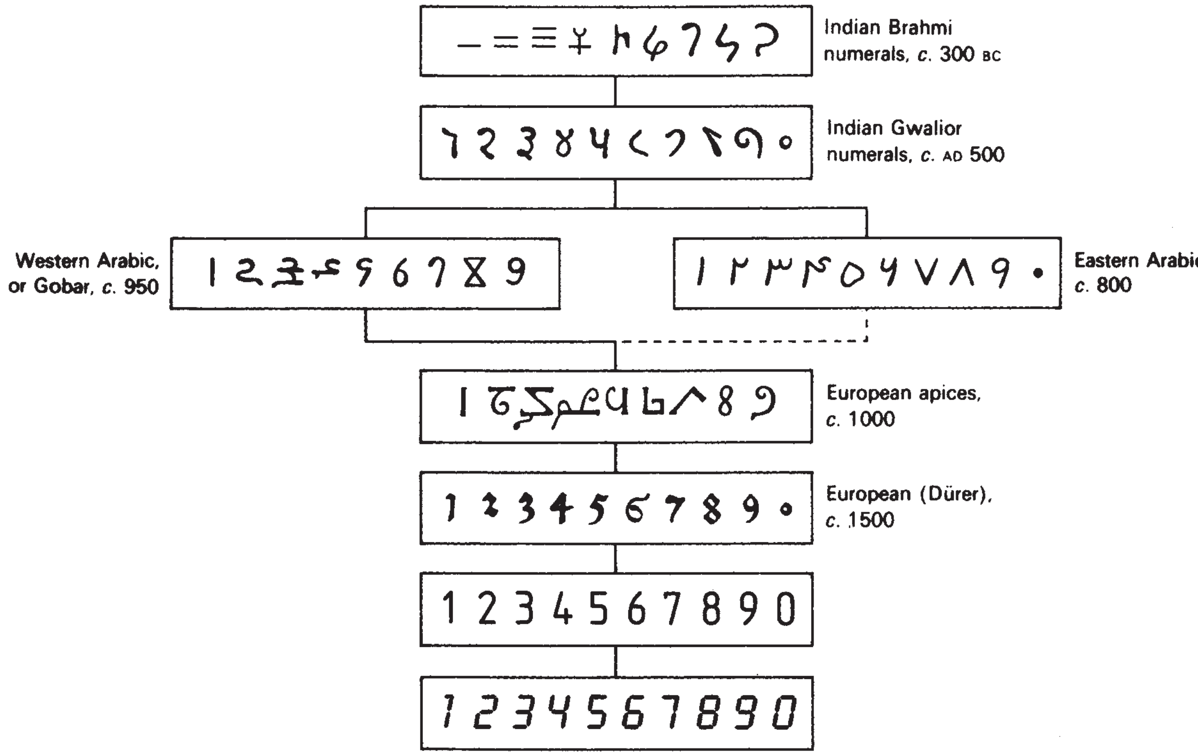

FIGURE 11.2: The evolution of present-day numerals (after Open University 1976, p. 53)

the Mediterranean, visiting Egypt, Syria, Greece, Sicily, and southern France, observing the various computational systems used by merchants, particularly in the Islamic world. He quickly recognized the enormous advantage of the Indian system, and introduced the new numerals with the following words:

> The nine Indian numerals are 9, 8, 7, 6, 5, 4, 3, 2, 1. With these nine and 0, which in Arabic is called *sifr*, any desired number can be written.

It was mainly through this work that the Indian numerals came to be widely known in Christian Europe. For a long time they were used alongside the Roman numerals. The change from the latter to the former was a slow process with a number of false starts, primarily because the abacus remained popular for carrying out calculations, and traders and others engaged in commercial activities were reluctant to adopt a new system that was difficult to comprehend. At times there were diktats from above to discourage the use of the new numerals. In 1299, for example, the city of Florence passed an ordinance prohibiting the use of the new numerals since they were more easily altered (e.g., by changing 0 to 6 or 9) than Roman numerals or numbers written out in words. As late as the end of the fifteenth century, the mayor of Frankfurt ordered his officials to refrain from calculating with Indian numerals. And even after the decimal numeral system was well established, Charles XII of Sweden (1682–1718) tried in vain to ban the decimal system and replace it with a base 64 system for which he devised sixty-four symbols! But these were all temporary setbacks. Once the contest between the "abacists" (those in favor of the use of the abacus or some mechanical device for calculation) and the "algorists" (those who favored the use of the new numerals) had been won by the latter, it was only a matter of time before the final triumph of the new numerals, with bankers, traders, and merchants adopting the system for their daily calculations.

Arithmetic in the Islamic World

Arithmetical Operations

The first systematic treatment of arithmetical operations is found in al-Khwarizmi's *Arithmetic*, in which he discusses the place-value system and rules for performing the four arithmetical operations. In later works, notably those of Ibn Labban and al-Uqlidisi referred to in the previous section,