MA 330 ASSIGNMENT # 4

Answers to problems may be handwritten.

(1) Around 1665, Newton proved that, for an arbitrary real number r, one has

$$(x+1)^r = \sum_{k=0}^{\infty} \frac{r(r-1)\cdots(r-k+1)}{k!} x^{r-k} = x^r + rx^{r-1} + \frac{r(r-1)}{2} x^{r-2} + \cdots .$$

Use this to produce a series approximation to the following values. Show each to at least the first four terms in each series.

(a) $\sqrt{11}$

(b) $(25)^{\frac{1}{3}}$

(2) Consider part 47 on page 94 of Newton's Method of Fluxions. Explain, in your own words, why the area of the pictured region AdB is given by the power series

$$\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{5}x^{\frac{5}{2}} - \frac{1}{28}x^{\frac{7}{2}} - \cdots$$

evaluated at $x = \frac{1}{4}$.

(3) Now consider part 48. Explain, in your own words, why the area of the pictured region AdB is $\frac{\pi}{24} - \frac{\sqrt{3}}{32}$.

(4) Putting together the two previous observations, explain in your own words how Newton obtains his approximation for π in part 49.