MA 330 ASSIGNMENT # 9

Problem 1:

On page 224 in Journey Through Genius, it is mentioned that Euler proved Fermat's claim that if a number is of the form $4k+3$, then it is not a sum of two squares. Prove Fermat's claim yourself, i.e., prove that a number of the form $4k+3$ is not expressible as a^2+b^2 for positive integers a and b.

HINT: Consider what happens when you evaluate a and b with different combinations of odd and even numbers (i.e., consider this mod 2).

Problem 2:

Prove that any number of the form $8k+7$ is not a sum of three squares, i.e., $8k+7$ is not equal to $a^2+b^2+c^2$ for any triple of positive integers a, b, and c.

HINT: If x, y, and z are integers, what are the possible remainders of $x^2+y^2+z^2$ after division by 8? What happens when you evaluate x, y, and z using different combinations of numbers of the form $8n+k$? (i.e., consider this mod 8.)

NOTE: In 1770, Lagrange proved that *every number is a sum of four squares.* This is AMAZING, and it is an example of how "high-dimensional" geometry (in other words, having lots of variables when you are doing algebra), can allow more freedom for solving problems than two or three dimensional geometry.