MA 391 ASSIGNMENT # 1

Answers to problems may be handwritten.

(1) Consider the voter preferences below.

Number of voters	18	12	10	9	4	2
First choice	A	B	C	D	E	E
Second choice	D	E	B	C	B	C
Third choice	E	D	E	E	D	D
Fourth choice	C	C	D	B	C	B
Fifth choice	B	A	A	A	A	A

Determine which candidate wins the election using 4 different election systems: plurality, sequential runoff, Borda count, and Condorcet. Which system do you think is best?

(2) In a plurality system, is it possible for a candidate to win the election, even if the majority of voters rank that candidate as their least favorite? Explain why or why not.

(3) Suppose you have a system for determining the winner of an election. Can you use it to determine a ranking of the candidates, from first to last?

(4) A *total ordering* of the candidates is a relation $\geq$ that satisfies the following properties:
   (a) (Anti-Symmetry) If $A \geq B$, then $B \ngeq A$.
   (b) (Totality) For any pair of candidates A and B, either $A \geq B$ or $B \geq A$.
   (c) (Transitivity) If $A \geq B$ and $B \geq C$, then $A \geq C$.

   Which of the relations below determine a total ordering of the candidates? Which of the three properties are satisfied, and which are not?
   (a) $A \geq_v B$ – Voter v prefers candidate A to candidate B.
   (b) $A \bigtriangledown B$ – Candidate A is either taller than or older than candidate B.
   (c) $A \vdash B$ – Candidate A is both taller and older than candidate B.
   (d) $A \heartsuit B$ – Candidate A loves candidate B.
   (e) $A \clubsuit B$ – Candidate A is precisely as smelly as candidate B.