

MA 391 ASSIGNMENT # 4

Answers to problems may be handwritten.

- (1) Consider a voting system for the six New England states where there are a total of 17 votes and 12 or more are required for passage. Massachusetts and Connecticut each get 4 votes, Maine and Rhode Island each get 3, New Hampshire gets 2, and Vermont gets 1. Calculate the Shapley-Shubik index and the Banzhaf indices of each state.

- (2) A yes-no voting system is said to be *monotone* if adding voters to a winning coalition again yields a winning coalition. Explain why, in a monotone system, every voter belongs to at least half of the winning coalitions. Show that a voter has no political power if and only if they belong to *exactly* half of the winning coalitions.

- (3) Suppose we have a yes-no voting system with 4 voters A, B, C, and D, and the winning coalitions are as follows: ABCD, ABC, ABD, ACD, BCD, AB, AD. Compute the Banzhaf index and Shapley-Shubik index of each voter.

- (4) Consider a voting system with 5 voters. One voter has 5 votes, one has 3 votes, and three have 1 vote each. Eight votes are needed for passage. Compute the Banzhaf indices for all 5 voters. Now, suppose that the first voter gives one of her votes to the second voter, so now 2 voters have 3 votes each and 3 voter have 1 vote each. Compute the Banzhaf power indices again. What do you notice?