MA 322

Assignment 11

1. Let L be the span of a nonzero vector $\vec{u}$ in $\mathbb{R}^2$. For $\vec{y}$ in $\mathbb{R}^2$ we say the reflection of y over L is

$$\text{refl}_L(\vec{y}) = 2\text{proj}_L\vec{y} - \vec{y}$$

   (a) Compute the projection of $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ over the line through the origin and the point $(1, 2)$.

   (b) Show that reflection is a linear transformation.

2. The vectors $\vec{u}_1 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$ and $\vec{u}_2 = \begin{bmatrix} 5 \\ -1 \\ 2 \end{bmatrix}$ are orthogonal. The vector $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ is not orthogonal to $\vec{u}_1$ and $\vec{u}_2$, but it is also not in the span of $\vec{u}_1$ and $\vec{u}_2$. Use these facts to construct a vector orthogonal to $\vec{u}_1$ and $\vec{u}_2$.

3. Let $\vec{y} = \begin{bmatrix} 3 \\ -1 \\ 1 \\ 13 \end{bmatrix}$, $\vec{u}_1 = \begin{bmatrix} 1 \\ -2 \\ -1 \\ 2 \end{bmatrix}$, and $\vec{u}_2 = \begin{bmatrix} -4 \\ 1 \\ 0 \\ 3 \end{bmatrix}$.

   (a) Find the point in the plane spanned by $\vec{u}_1$ and $\vec{u}_2$ that is closest to $\vec{y}$.

   (b) What is the distance from $\vec{y}$ to the plane spanned by $\vec{u}_1$ and $\vec{u}_2$?

4. Find an orthogonal basis for the column space of the matrix $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$.

5. Let $A = \begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}$

   (a) Find an orthogonal basis for the column space of A.

   (b) Find the projection of $\vec{b} = \begin{bmatrix} 7 \\ 2 \\ 7 \\ 0 \end{bmatrix}$ onto the column space of A.

   (c) Find the least squares solution to $A\vec{x} = \vec{b}$.