

Assignment 13

1. Is the set of all polynomials $\mathbf{p}(x)$ in \mathbb{P}_n so that $\mathbf{p}(0) = 0$ a subspace of \mathbb{P}_n ? *Justify!*
2. Fix a 3×2 matrix F . If H is the set of all matrices A in $M_{2 \times 4}$ so that $FA = 0$, is H a subspace of $M_{2 \times 4}$? *Justify!*
3. Let $T: \mathbb{P}_2 \rightarrow \mathbb{M}_{2 \times 2}$ be the transformation $T(p) = \begin{bmatrix} p(0) & p(-1) \\ p(1) & p(2) \end{bmatrix}$.
 - (a) Show T is linear.
 - (b) What is the kernel of T ?
 - (c) What is the range of T ?
4. Let $S: \mathbb{P}_3 \rightarrow \mathbb{M}_{2 \times 2}$ be the transformation $S(p) = \begin{bmatrix} p(0) & p(-1) \\ p(1) & p(2) \end{bmatrix}$. (The only difference between S and T is the type of input.)
 - (a) What is the kernel of S ?
 - (b) What is the range of S ?
5. Let C be the vector space of all continuous real valued functions of a single real variable. There is a linear transformation $D: C \rightarrow C$ defined by $D(f)$ is the derivative of f . What is the kernel of D ?
6. Let $\vec{p}_1(t) = 1 + t$, $\vec{p}_2(t) = 1 - t$, $\vec{p}_3(t) = 2$.
 - (a) Are $\{\vec{p}_1(t), \vec{p}_2(t), \vec{p}_3(t)\}$ a linearly independent set?
 - (b) Find a basis for the span of $\{\vec{p}_1(t), \vec{p}_2(t), \vec{p}_3(t)\}$.
 - (c) What is the coordinate vector of $4t + 3$ relative to the basis you chose in (b)?
7. Is $\{t, \sin(t), \cos(t)\}$ a linearly independent set of vectors? (Hint: functions are the same only when the values for all choices of t are the same.)