Assignment 14

1. The set $\mathcal{B} = \{1 - t^2, t - t^2, 2 - 2t + t^2\}$ is a basis for $\mathbb{P}_2$.

    (a) What is the coordinate vector of $6 + 2t - 12t^2$ with respect to $\mathcal{B}$?

    (b) If $[p(x)]_{\mathcal{B}} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ what is $p(x)$?

2. Let $T \colon \mathbb{P}_2 \to \mathbb{M}_{2\times 2}$ be the transformation $T(p) = \begin{bmatrix} p(0) & p(-1) \\ p(1) & p(2) \end{bmatrix}$. What is the matrix of T relative to the standard bases for $\mathbb{P}_2$ and $\mathbb{M}_{2\times 2}$?

3. Let $S \colon \mathbb{P}_3 \to \mathbb{M}_{2\times 2}$ be the transformation $S(p) = \begin{bmatrix} p(0) & p(-1) \\ p(1) & p(2) \end{bmatrix}$. (The only difference between S and T is the type of input.) What is the matrix of S relative to the standard bases for $\mathbb{P}_3$ and $\mathbb{M}_{2\times 2}$?

4. Let $T \colon \mathbb{P}_2 \to \mathbb{R}^2$ be given by $T(p(x)) = \begin{bmatrix} \int_{-1}^{0} p(x)\,dx \\ \int_{0}^{1} p(x)\,dx \end{bmatrix}$.

    (a) Compute $T(x^2), T(x), T(1)$.
    (b) What is the matrix of T with respect to the standard bases of $\mathbb{P}_2$ and $\mathbb{R}^2$?

5. Let V be the subspace of the vector space of real valued continuous functions of a single real variable generated by $\cos(x)$ and $\sin(x)$. Let $T \colon V \to V$ be defined by $T(f(x)) = f'(x) + f''(x)$. What is the matrix of T relative to the basis $\{\cos(x), \sin(x)\}$?

6. Find the change of coordinates matrix from $\mathcal{B} = \left\{ \begin{bmatrix} -1 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ -5 \end{bmatrix} \right\}$ to $\mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$.

7. In $\mathbb{P}_2$ find the change of coordinates matrix from $\mathcal{B} = \{1 - 3t^2, 2 + t - 5t^2, 1 + 2t\}$ to the standard basis.