MA 322

Assignment 7

1. Let $\mathcal{B} = \left\{ \begin{bmatrix} -3 \\ 1 \\ -4 \end{bmatrix}, \begin{bmatrix} 7 \\ 5 \\ -6 \end{bmatrix} \right\}$.

   (a) Find the coordinate vector of $\begin{bmatrix} 4 \\ 6 \\ -10 \end{bmatrix}$ relative to $\mathcal{B}$.

   (b) Find the coordinate vector of $\begin{bmatrix} 11 \\ 0 \\ 7 \end{bmatrix}$ relative to $\mathcal{B}$.

2. The matrices

$$A = \begin{bmatrix} 1 & -2 & 9 & 5 & 4 \\ 1 & -1 & 6 & 5 & -3 \\ -2 & 0 & -6 & 1 & -2 \\ 4 & 1 & 9 & 1 & -9 \end{bmatrix} \qquad \begin{bmatrix} 1 & -2 & 9 & 5 & 4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

   are row equivalent. Find bases for Col A and Nul A and give the dimensions of these subspaces.

3. Compute the determinant of

$$\begin{bmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -4 & -3 & 5 \\ 2 & 0 & 3 & 5 \end{bmatrix}$$

   by first expanding along the first row (at every stage) and then by expanding along whatever row or column requires the fewest computations.

4. If A is a 2×2 matrix, what is $\det(4A)$ in terms of $\det(A)$?

5. Use row operations to compute the following determinant

$$\begin{vmatrix} 3 & 3 & -3 \\ 3 & 4 & -4 \\ 2 & -3 & -5 \end{vmatrix}$$

6. Explain why a square matrix A with $\det(A^3) = 0$ cannot be invertible.