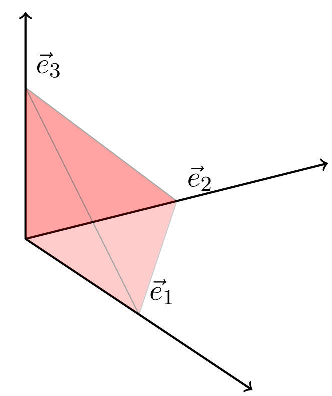
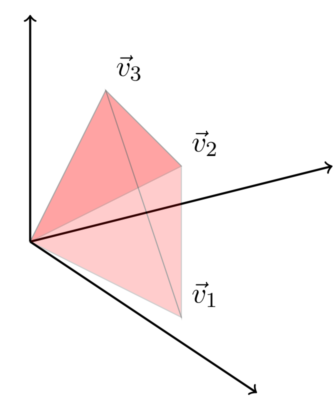
MA 322

Assignment 8

1. Let $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by the matrix $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ where a, b and c are positive numbers. Let S be the unit ball. Show that $T(S)$ is bounded by the ellipsoid with the equation $\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1$ and find the volume of this ellipsoid.

2. Let A be the tetrahedron in $\mathbb{R}^3$ with vertices $\vec{0}$, $\vec{e}_1$, $\vec{e}_2$, and $\vec{e}_3$. Let B be the tetrahedron in $\mathbb{R}^3$ with vertices $\vec{0}$, $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$.





   (a) Find a transformation T so that $T(A) = B$.

   (b) Find the volume of B using the fact that the volume of A is $\frac{1}{3}$(area of the base)(height).

3. Use the definition of eigenvalue to fined the eigenvalues of the matrix

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & -3 \end{bmatrix}$$

4. Show that if A^2 is the zero matrix then the only eigenvalue of A is 0.

5. Find the eigenvalues and eigenspaces for the matrix

$$\begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$$

6. Show that A and A^T have the same characteristic polynomial.