MA 665 EXERCISES 1

(1) Show that the ring homomorphism $\mathbb{Z} \hookrightarrow \mathbb{Q}$ is both a monomorphism and an epimorphism in the category of commutative rings with unit, though it is not an isomorphism.

(2) Show that if $g \circ f$ is a monomorphism, then so is f. Similarly, show that if $g \circ f$ is an epimorphism, then so is g. Conclude that any isomorphism is both a monomorphism and an epimorphism. (Note that this question is asking about the categorical definition of monomorphisms and epimorphisms, so in particular you cannot prove it by plugging in elements.)

(3) If $\mathcal{G}$ and $\mathcal{H}$ are groups regarded as categories with one object, characterize functors from $\mathcal{G}$ to $\mathcal{H}$.