MA 665 EXERCISES 5

(1) Let R be a ring. Prove that every R-module is projective if and only if every R-module is injective.

(2) Let R be a commutative ring. Prove that $R[x]$ is a flat R-module.

(3) Let M_1 and M_2 be R-modules. Show that $M_1 \oplus M_2$ is an injective R-module if and only if both M_1 and M_2 are injective R-modules. Conclude that, if R is a PID that is not a field, then no nonzero finitely generated R-module is injective.