SERIES

October 02, 2006

1. Determine if each of the following sequences converges or diverges. Find the limit if requested.

   (a) $\lim\limits_{n\to\infty} \dfrac{n!}{n^n}$

   (b) Verify that $\lim\limits_{n\to\infty} \sqrt[n]{a^n + b^n} = \max(a, b)$

   (c) $\lim\limits_{n\to\infty} n - \sqrt{n+a}\sqrt{n+b}$

   (d) $\lim\limits_{n\to\infty} \dfrac{(-1)^n \sqrt{n}\sin(n^n)}{n+1}$

   (e) $\lim\limits_{n\to\infty} \dfrac{a^n - b^n}{a^n + b^n}$

   (f) $\lim\limits_{n\to\infty} nc^n, \; |c| < 1$

   (g) $\lim\limits_{n\to\infty} \dfrac{2^{n^2}}{n!}$

   (h) $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2a_n}$. Find $\lim\limits_{n\to\infty} a_n$

2. Decide whether each of the following infinite series is convergent or divergent. What test did you use?

   (a) $\displaystyle\sum_{n=1}^{\infty} \dfrac{\sin n\theta}{n^2}$

   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \dfrac{\log n}{n}$

   (c) $\displaystyle\sum_{n=2}^{\infty} \dfrac{1}{\sqrt[3]{n^2 - 1}}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \dfrac{n^2}{n!}$

   (e) $\displaystyle\sum_{n=1}^{\infty} \dfrac{\log n}{n}$

   (f) $\displaystyle\sum_{n=2}^{\infty} \dfrac{1}{\log n}$

   (g) $\displaystyle\sum_{n=2}^{\infty} \dfrac{1}{(\log n)^k}, \; k < n$

   (h) $\displaystyle\sum_{n=1}^{\infty} \dfrac{1}{(\log n)^n}$

   (i) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \dfrac{1}{(\log n)^n}$

   (j) $\displaystyle\sum_{n=1}^{\infty} \dfrac{n^2}{n^3 + 1}$

   (k) $\displaystyle\sum_{n=1}^{\infty} \sin \dfrac{1}{n}$

   (l) $\displaystyle\sum_{n=2}^{\infty} \dfrac{1}{n \log n}$

   (m) $\displaystyle\sum_{n=1}^{\infty} \dfrac{1}{n(\log n)^2}$

   (n) $\displaystyle\sum_{n=1}^{\infty} \dfrac{1}{n^2(\log n)}$

   (o) $\displaystyle\sum_{n=1}^{\infty} \dfrac{1}{n^{1+1/n}}$