ASSIGNMENT 10

20-November-2006

1. Prove that if f is uniformly continuous on a bounded set S, then f is a bounded function on S, i.e., there is an $M > 0$, $M \in \mathbb{R}$, so that $|f(x)| \leq M$ for all $x \in S$.
   HINT: Try proof by contradiction.

2. The Mean Value Theorem states that if f is continuous on $[a, b]$ and differentiable on (a, b), then there is at least one $x \in (a, b)$ so that

$$f'(x) = \frac{f(b) - f(a)}{b - a}.$$

   (a) Use the *Mean Value Theorem* to prove that

$$|\sin x. - \sin y| \leq |x - y|$$

   for $x, y \in \mathbb{R}$.

   (b) Show that $\sin x$ is uniformly continuous on $\mathbb{R}$.