ASSIGNMENT 3

18-September-2006

1. Let (a,b) and (c,d) be any two open intervals in the real line.
   (a) Find a one-to-one function that maps $(0,1)$ to $(-1,1)$.
   (b) Find a one-to-one function from (a,b) to (c,d). You must show that it is one-to-one.
   (c) Prove that any two open intervals in the real line have the same cardinality.
2. Let $f(x) = 1/(1+x)$. What is
   (a) $f(f(x))$ (for which x does this make sense?)?
   (b) $f\left(\frac{1}{x}\right)$?
   (c) $f(cx)$?
   (d) $f(x+y)$?
   (e) $f(x) + f(y)$?
   (f) For which numbers c is there a number x such that $f(cx) = f(x)$?
   (g) For which numbers c is it true that $f(cx) = f(x)$ for two different numbers x?
3. For which numbers a, b, c, d will the function
$$f(x) = \frac{ax+b}{cx+d}$$
   satisfy $f(f(x)) = x$ for all x?
4. Suppose that H is a function.
   (a) Suppose that y is a number such that $H(H(y)) = y$, what is
$$\underbrace{H(H(H(\cdots(H(y)\cdots))))}_{20 \text{ times}}?$$
   (b) Same question if 20 is replaced by 21.
   (c) Same question if $H(H(y)) = H(y)$.
5. Let f and g be functions $f, g\colon \mathbb{R} \to \mathbb{R}$.
   (a) Determine whether $f+g$ is even, odd, or neither in the four cases obtained by choosing f even or odd and g even or odd.

$f+g$	f even	f odd
g even		
g odd		

   (b) Do the same for $f \cdot g$.
   (c) Do the same for $f \circ g$.
6. Let f, g, and h be functions from the reals to the reals. Prove or give a counterexample to each of the following.
   (a) $f \circ (g+h) = f \circ g + f \circ h$
   (b) $(g+h) \circ f = g \circ f + h \circ f$
   (c) $\dfrac{1}{f \circ g} = \dfrac{1}{f} \circ g$
   (d) $\dfrac{1}{f \circ g} = f \circ \dfrac{1}{g}$