

ASSIGNMENT 4

25-September-2006

1. Find the limits of the following sequences

(a) $a_n = \sqrt{n^2 + 1} - n$.
[HINT] Show that $a_n = \frac{1}{\sqrt{n^2+1}+n}$.

(b) $b_n = \sqrt{n^2 + n} - n$.

(c) $c_n = \sqrt{4n^2 + n} - 2n$.

2. Suppose that $\lim x_n = 3$, $\lim y_n = 7$ and that all y_n are nonzero. Determine the following limits:

(a) $\lim(x_n + y_n)$ (b) $\lim \frac{3y_n - x_n}{y_n^2}$

3. Let $a_1 = 1$ and for $n \geq 1$ let $a_{n+1} = \sqrt{a_n + 1}$.

(a) List the first five terms of $\{a_n\}$.

(b) It turns out that $\{a_n\}$ converges. Assume that this is true and show that the limit is $\frac{1}{2}(1 + \sqrt{5})$.

4. Let $a_1 = 1$ and $a_{n+1} = \frac{1}{3}(a_n + 1)$ for $n \geq 1$.

(a) Find a_2 , a_3 , a_4 and a_5 .

(b) Use induction to show that $a_n > \frac{1}{2}$ for all n .

(c) Show that $\{a_n\}$ is a nonincreasing sequence.

(d) Find $\lim a_n$.

5. For each of the following sequences find the $\text{glb}\{a_n\}$, $\text{lub}\{a_n\}$, $\limsup\{a_n\}$, and $\liminf\{a_n\}$.

(a) $\{(-1)^n\}_{n=0}^{\infty}$

(b) $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$

(c) $\{(-1)^n n\}_{n=0}^{\infty}$

6. Let $\{a_n\}$ and $\{b_n\}$ be the following sequences that repeat in cycles of four.

$$\{a_n\} = \{0, 1, 2, 1, 0, 1, 2, 1, 0, 1, 2, 1, 0, 1, 2, 1, 0, \dots\}$$

$$\{b_n\} = \{2, 1, 1, 0, 2, 1, 1, 0, 2, 1, 1, 0, 2, 1, 1, 0, 2, \dots\}$$

Find

(a) $\liminf a_n + \liminf b_n$

(b) $\liminf(a_n + b_n)$

(c) $\liminf a_n + \limsup b_n$

(d) $\limsup(a_n + b_n)$

(e) $\limsup a_n + \limsup b_n$

(f) $\liminf a_n b_n$

(g) $\limsup a_n b_n$